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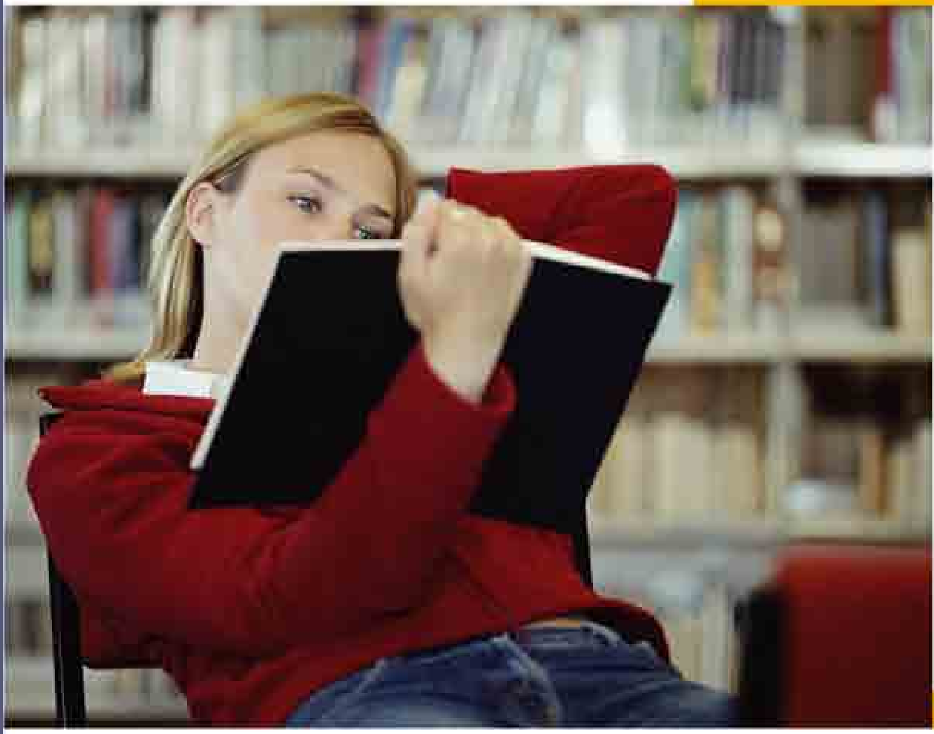

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CHAPTER

9 Conic Sections

If a gun is fired upward with an initial velocity of 288 ft/sec, the bullet's height h after t seconds is given by the equation $h = 288t - 16t^2$. Find the maximum height attained by the bullet.

In chapter 7, we graphed first-degree equations in two variables of the form $ax + by = c$, which we called linear equations since the graph was a straight line. In this chapter, we will consider the graphs of second-degree equations in which one or more terms are of second-degree. These graphs result from slicing a cone with a plane, called the cutting plane, as shown in figure 9–1. For that reason, the graphs are called the **conic sections.**

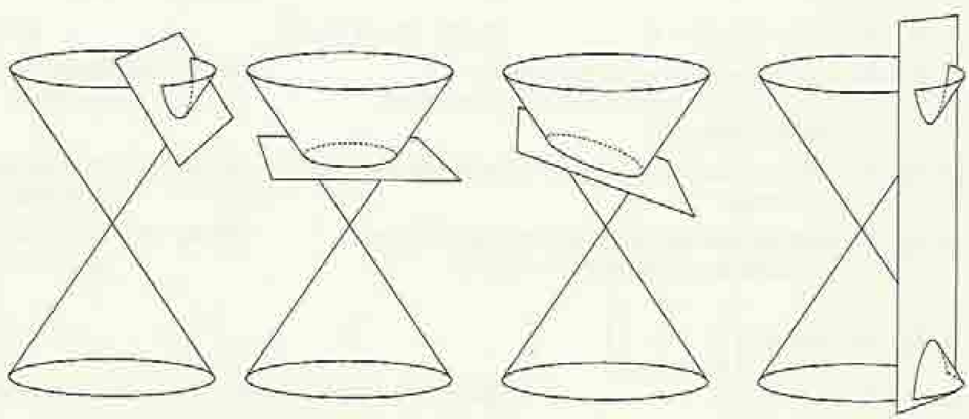


Figure 9–1

9–1 ■ The parabola

If we assign coordinates to the cutting plane, each conic section can be described by a second-degree (quadratic) equation. The graph of the second-degree equation $y = ax^2 + bx + c$ ($a \neq 0$) is a figure called a **parabola.**

Consider the quadratic equation in two variables

$$y = x^2 - 4$$

If we choose the values for x in the set $\{-3,-2,-1,0,1,2,3\}$ and compute the corresponding values of y, the ordered pairs we obtain are

$$(-3,5), (-2,0), (-1,-3), (0,-4), (1,-3), (2,0), \text{ and } (3,5)$$

Plotting these ordered pairs and connecting the points with a smooth curve, we obtain the graph, a *parabola*, shown in figure 9–2. This parabola is the graph of the equation $y = x^2 - 4$.

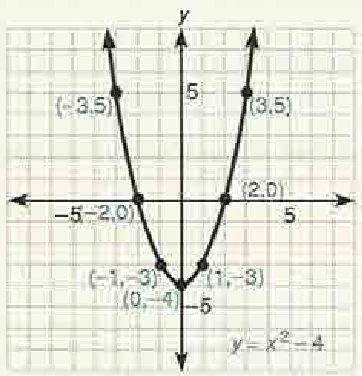


Figure 9–2

Similarly, consider the graph of the equation $y = -x^2 + 1$. Choosing values of x and then finding the corresponding values of y, we get the results shown in this table of related values.

x	-2	-1	0	1	2
y	-3	0	1	0	-3

Plot the points and connect them with a smooth curve to obtain the parabola shown in figure 9–3.

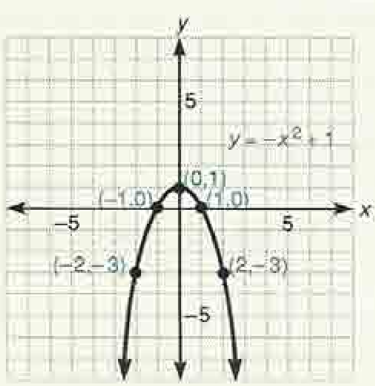


Figure 9–3

As we examine the parabolas with equations of the form $y = ax^2 + bx + c$ in figures 9–2 and 9–3, we observe some of the basic properties of the parabola.

1. For each x, there is a unique value of y.
2. There is a *lowest* point [(0, −4) in figure 9–2] or a *highest* point [(0,1) in figure 9–3] of each graph. We call this the **vertex** of the parabola.
3. A vertical line through the vertex, in this case the y-axis, divides the parabola into two identical (mirror-image) parts. This line is called the **axis of symmetry.**
4. For each point on the parabola to the right of the axis of symmetry, there is a corresponding point to the left of the axis of symmetry.
5. When the coefficient of the squared term is
   a. positive, as in $y = x^2 - 4$, the parabola opens upward and the vertex is the lowest point of the graph;
   b. negative, as in $y = -x^2 + 1$, the parabola opens downward and the vertex is the highest point of the graph.

Equation for a parabola with vertical axis of symmetry

$$y = ax^2 + bx + c$$

where a, b, and c are real numbers, $a \neq 0$.

Now let us consider the graph of the general quadratic equation $y = ax^2 + bx + c$. We could choose a number of values for x and obtain the corresponding values for y as we did before. However the parabola has certain special points (when they exist) that we can use to sketch the graph when we know the general shape of the curve. These points are (1) the vertex and (2) the x- and y-intercepts. These points, together with the previously observed characteristics of the parabola, enable us to obtain a reasonably accurate sketch of the curve.

The vertex

To obtain the coordinates of the vertex, we use the procedure of completing the square that we learned in chapter 6. Completing the square in the right member, we can write the equation $y = ax^2 + bx + c$ in the form

$$y = a(x - h)^2 + k \qquad (a \neq 0)$$

where the point (h,k) is the vertex of the parabola and the line $x = h$ is the axis of symmetry. To show this is true, when $a > 0$, then $a(x - h)^2 \geq 0$ and the least value of y occurs when $a(x - h)^2 = 0$, or when $x = h$. If $x = h$, then $y = a(h - h)^2 + k = a \cdot 0 + k = k$. The lowest point (the vertex) is at (h,k). A similar argument can be made when $a < 0$, where the vertex (h,k) is the highest point.

Note If $a > 0$, the parabola opens upward, and if $a < 0$, the parabola opens downward.

Forms of the equation of a parabola

Given a parabola that opens upward when $a > 0$ and downward when $a < 0$,

1. The *general form* of the equation of the parabola is $y = ax^2 + bx + c$.
2. The *standard form* of the equation of the parabola is $y = a(x - h)^2 + k$ where the point (h,k) is the vertex of the parabola.

Example 9–1 A

Determine the coordinates of the vertex of the parabola. State whether the parabola opens upward or downward.

1. $y = (x - 2)^2 + 3$
Since $h = 2$ and $k = 3$, the vertex is the point $(2,3)$ and since $a = 1$, $a > 0$, the parabola opens upward.

2. $y = -3(x + 4)^2 - 6$
The equation is not exactly in the form $y = a(x - h)^2 + k$, so we rewrite the equation as

$$y = -3[x - (-4)]^2 + (-6)$$

Now $h = -4$ and $k = -6$, so the vertex is the point $(-4,-6)$. Since $a = -3$, $a < 0$, the parabola opens downward.

3. $y = x^2 + 4x - 12$
Rewrite the equation in the form

$$y = (x^2 + 4x + ?) - 12 - (?)$$

Complete the square inside parentheses as we learned in section 6–2.

$$y = (x^2 + 4x + 4) - 12 - (4)$$

Note To complete the square, we added 4. Thus, we must *subtract* 4 so that the equation is not changed.

Write the trinomial as a binomial square and combine -12 and -4.

$$y = (x + 2)^2 - 16$$
$$y = (x - (-2))^2 + (-16) \qquad \text{Write in form } (x - h)^2 + k$$

Thus, $h = -2$ and $k = -16$ so the vertex is the point $(-2,-16)$. Since $a = 1$, $a > 0$, the parabola opens upward.

▶ ***Quick check*** Determine the coordinates of the vertex of the parabola $y = x^2 + 2x - 8$.

The *x*-intercept(s)

There can be two, one, or no x-intercepts on the graph of a quadratic equation $y = ax^2 + bx + c$. The x-intercept(s) (if there are any) are found by replacing y with 0 and solving for x. This is the same as we did with linear equations.

The y-intercept

There will always be *one* y-intercept in the graph of $y = ax^2 + bx + c$. As with linear equations, let $x = 0$ and solve for y. Given

$$\begin{aligned} y &= ax^2 + bx + c \\ &= a(0)^2 + b(0) + c \qquad \text{Replace } x \text{ with } 0 \\ &= c \end{aligned}$$

The y-intercept of the quadratic equation in general form $y = ax^2 + bx + c$ is always the point $(0,c)$.

Example 9–1 B

Find the x- and y-intercepts of the parabola $y = x^2 + 5x - 14$.

1. Let $y = 0$.

$$\begin{aligned} (0) &= x^2 + 5x - 14 && \text{Replace } y \text{ with } 0 \\ 0 &= (x + 7)(x - 2) && \text{Factor the right member} \\ x + 7 = 0 &\quad \text{or} \quad x - 2 = 0 && \text{Set each factor equal to } 0 \\ x = -7 &\qquad\qquad x = 2 && \text{Solve for } x \end{aligned}$$

The x-intercepts are the points $(-7,0)$ and $(2,0)$.

2. Since $c = -14$, the y-intercept is the point $(0,-14)$.

▶ ***Quick check*** Find the x- and y-intercepts of the parabola $y = x^2 + 2x - 8$.

We now outline the procedure for graphing a quadratic equation of the form $y = ax^2 + bx + c$.

To graph the quadratic equation $y = ax^2 + bx + c$

1. Determine whether the parabola opens upward ($a > 0$) or downward ($a < 0$).
2. Find the y-intercept and any x-intercept(s).
3. Determine the coordinates of the vertex by completing the square to get the equation in the form $y = (x - h)^2 + k$. The vertex is the point (h,k).
4. Find and plot other points to the right and left of the axis of symmetry, $x = h$, as needed.

Example 9–1 C

Sketch the graphs of the following equations using the vertex and the x- and y-intercepts.

1. $y = x^2 - 4x - 5$

a. Since $a = 1$, which is positive, the parabola opens upward.

b. Let $y = 0$, then

$$\begin{aligned} (0) &= x^2 - 4x - 5 && \text{Replace } y \text{ with } 0 \\ 0 &= (x - 5)(x + 1) && \text{Factor the right member} \\ x - 5 = 0 &\text{ or } x + 1 = 0 && \text{Set each factor equal to } 0 \\ x = 5 &\qquad x = -1 \end{aligned}$$

The x-intercepts are $(5,0)$ and $(-1,0)$.

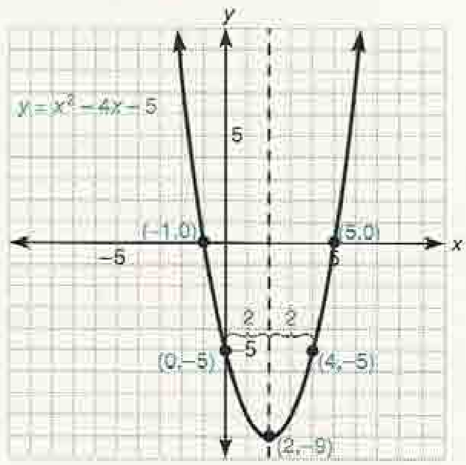
c. Since $c = -5$, the y-intercept is $(0,-5)$.

d. Given $y = x^2 - 4x - 5$

$$\begin{aligned} &= (x^2 - 4x + \quad) - 5 - (\quad) \\ &= (x^2 - 4x + 4) - 5 - (4) && \text{Complete the square} \\ &= (x - 2)^2 - 9 && \text{Write binomial square} \\ &= (x - 2)^2 + (-9) && \text{Write in form } y = a(x - h)^2 + k \end{aligned}$$

The vertex is the point $(2,-9)$

e. Using the axis of symmetry, $x = 2$, choose point $(4,-5)$ that is symmetric with the y-intercept $(0,-5)$.



2. $y = -x^2 + 2x + 3$

a. Since $a = -1$ (negative), the parabola opens downward.

b. Let $y = 0$.

$$\begin{aligned} (0) &= -x^2 + 2x + 3 && \text{Replace } y \text{ with } 0 \\ 0 &= x^2 - 2x - 3 && \text{Multiply each member by } -1 \\ 0 &= (x - 3)(x + 1) && \text{Factor the right member} \\ x - 3 = 0 &\text{ or } x + 1 = 0 && \text{Set each factor equal to } 0 \\ x = 3 &\qquad x = -1 \end{aligned}$$

The x-intercepts are $(3,0)$ and $(-1,0)$.

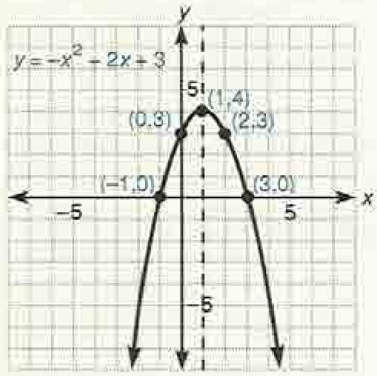
c. Since $c = 3$, the y-intercept is $(0,3)$.

d. Given $y = -x^2 + 2x + 3$

$$\begin{aligned} &= -1(x^2 - 2x \quad\;) + 3 - (\quad) && \text{Factor } -1 \text{ to obtain } x^2 \\ &= -1(x^2 - 2x + 1) + 3 - (-1) && \text{Complete the square} \\ &= -1(x - 1)^2 + 4 \end{aligned}$$

The vertex is the point $(1,4)$.

e. The point (2,3) is symmetric to the y-intercept (0,3) about the axis of symmetry $x = 1$.



The coordinates of the vertex can also be found this way:

1. Let $x = -\dfrac{b}{2a}$.
2. Replace x with this value in the equation and solve for y.

This method is demonstrated in example 3.

3. $y = -2x^2 - 4x - 3$

a. Since $a = -2$ (negative), the parabola opens downward.

b. Let $y = 0$.

$$(0) = -2x^2 - 4x - 3 \quad \text{Replace } y \text{ with } 0$$
$$0 = 2x^2 + 4x + 3 \quad \text{Multiply each member by } -1$$

We cannot factor $2x^2 + 4x + 3$, so we use the quadratic formula to solve for x.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(3)}}{2(2)} \quad \text{Replace } a \text{ with } 2, b \text{ with } 4, \text{ and } c \text{ with } 3$$
$$= \frac{-4 \pm \sqrt{-8}}{4} \quad \text{Simplify}$$

Since $\sqrt{-8}$ is not a real number, there are no x-intercepts. The graph does not cross the x-axis because the rectangular coordinate system graphs ordered pairs of *real numbers.*

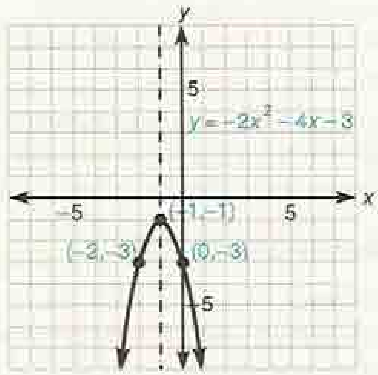
c. Since $c = -3$, the y-intercept is $(0,-3)$.

d. Since $a = -2$ and $b = -4$ in the original equation,

$$(1)\ x = -\frac{b}{2a} = -\frac{(-4)}{2(-2)} = -1 \quad \text{Replace } a \text{ with } -2 \text{ and } b \text{ with } -4$$
$$(2)\ y = -2(-1)^2 - 4(-1) - 3 \quad \text{Replace } x \text{ with } -1$$
$$= -2 + 4 - 3 \quad \text{Simplify}$$
$$= -1$$

The vertex is the point $(-1,-1)$.

e. The point $(-2,-3)$ is symmetric to the y-intercept $(0,-3)$ about the axis of symmetry $x = -1$.



▶ ***Quick check*** Sketch the graph of $y = x^2 + 4x + 3$. ■

When considering the quadratic equation $y = ax^2 + bx + c$, we see that the graph has a *low point* (or *minimum value* of y) when $a > 0$ and a *high point* (or *maximum value* of y) when $a < 0$. We can use these values of y to answer questions in a physical situation.

■ Example 9–1 D

1. A projectile is fired upward from the ground so that its distance s in feet above the ground t seconds after firing is given by $s = -16t^2 + 96t$. Find the maximum height it will reach and the number of seconds it takes to reach that height.

   Since $a = -16$, the graph will have a high point, or maximum value of s, that occurs at t. We must find the vertex by completing the square in the right member.

   $$\begin{aligned} s &= -16t^2 + 96t \\ &= -16(t^2 - 6t + \quad) - (\quad) && \text{Factor } -16 \text{ from each term} \\ &= -16(t^2 - 6t + 9) - (-144) && \text{Complete the square} \\ &= -16(t - 3)^2 + 144 \end{aligned}$$

   The vertex of the parabola is (3,144) and the projectile will reach the maximum height s of 144 feet in $t = 3$ seconds after firing.

2. Harry has 48 feet of fencing to use to fence off a rectangular area behind his house for his dog Nappy. What are the dimensions of the largest area that he can fence off with his 48 feet of fencing?

   Let x = the length of one of the equal sides. Then $48 - 2x$ = the length of the side opposite the house. (Harry has 48 feet of fencing and the other two sides are x in length.)

   Using area (A) = length (ℓ) × width (w), where width $(w) = x$ and length $(\ell) = 48 - 2x$, we have the equation.

   $A = x(48 - 2x) = -2x^2 + 48x$

Completing the square in the right member,

$$\begin{aligned} A &= -2x^2 + 48x \\ &= -2(x^2 - 24x + \quad) - (\quad) && \text{Factor } -2 \text{ from each term} \\ &= -2(x^2 - 24x + 144) - (-288) && \text{Complete the square} \\ &= -2(x - 12)^2 + 288 \end{aligned}$$

The vertex is the point (12,288) and the maximum area is 288 square feet when $x = 12$. Then $48 - 2x = 48 - 2(12) = 24$. The dimensions of the rectangle are 12 feet by 24 feet. ■

Mastery points

Can you

- Find the x- and y-intercepts of a quadratic equation in two variables?
- Find the coordinates of the vertex of the parabola?
- Sketch the graph of a quadratic equation in two variables?
- Find the maximum and minimum values of a quadratic equation of the form $y = ax^2 + bx + c$, $a \neq 0$?

Exercise 9–1

Find the coordinates of the vertex of the parabola for the graph of each quadratic equation. See example 9–1 A.

Example $y = x^2 + 2x - 8$

Solution Given $y = x^2 + 2x - 8$

$$\begin{aligned} y &= (x^2 + 2x + \quad) - 8 - (\ ) \\ y &= (x^2 + 2x + 1) - 8 - (1) && \text{Complete the square} \\ y &= (x + 1)^2 - 9 && \text{Write the square of a binomial} \\ y &= [x - (-1)]^2 + (-9) && \text{Write in form } y = a(x - h)^2 + k \end{aligned}$$

The vertex is the point $(-1,-9)$.

1. $y = (x - 3)^2 + 4$
2. $y = 3(x + 1)^2 - 7$
3. $y = x^2 - 16$
4. $y = x^2 + 4$
5. $y = (x - 5)^2$
6. $y = (x + 6)^2$
7. $y = x^2 + 4x - 5$
8. $y = x^2 - 6x + 5$
9. $y = -x^2 + 2x + 3$
10. $y = -x^2 + 3x - 2$
11. $y = 2x^2 - 7x + 3$
12. $y = -5x^2 + 6x - 1$

Find the x- and y-intercepts of the following parabolas. See example 9–1 B.

Example $y = x^2 + 2x - 8$

Solution 1. Let $y = 0$.

$$\begin{aligned} (0) &= x^2 + 2x - 8 && \text{Replace } y \text{ with } 0 \\ 0 &= (x + 4)(x - 2) && \text{Factor the right member} \\ x + 4 = 0 &\text{ or } x - 2 = 0 && \text{Set each factor equal to } 0 \\ x = -4 & \qquad x = 2 \end{aligned}$$

The x-intercepts are the points $(-4,0)$ and $(2,0)$.

2. Since $c = -8$, the y-intercept is the point $(0,-8)$

$\boxed{13.}$ $y = (x - 3)^2 + 4$

14. $y = 3(x + 1)^2 - 7$

15. $y = x^2 - 16$

16. $y = x^2 + 4$

17. $y = (x - 5)^2$

$\boxed{18.}$ $y = (x + 6)^2$

19. $y = x^2 + 4x - 5$

20. $y = x^2 - 6x + 5$

$\boxed{21.}$ $y = -x^2 + 2x + 3$

22. $y = -x^2 + 3x - 2$

23. $y = 2x^2 - 7x + 3$

24. $y = -5x^2 - 6x - 1$

Sketch the graph of each given quadratic equation using the intercepts and the coordinates of the vertex where possible. See example 9–1 C.

Example $y = x^2 + 4x + 3$

Solution

1. Since $a = 1$ (positive), the parabola opens upward.
2. Since $c = 3$, the y-intercept is $(0,3)$.
3. Let $y = 0$.

$0 = x^2 + 4x + 3$ — Replace y with 0

$0 = (x + 3)(x + 1)$ — Factor the right member

$x + 3 = 0$ or $x + 1 = 0$ — Set each factor equal to 0

$x = -3 \qquad x = -1$

The x-intercepts are $(-3,0)$ and $(-1,0)$.

4. Given $y = x^2 + 4x + 3$

$y = (x^2 + 4x + \quad) + 3 - (\quad)$

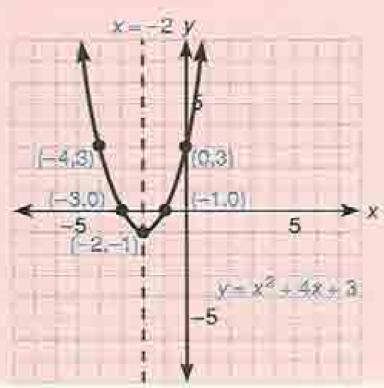
$y = (x^2 + 4x + 4) + 3 - (4)$ — Complete the square

$y = (x + 2)^2 - 1$ — Write as the square of a binomial

$y = [x - (-2)]^2 + (-1)$ — Write in form $y = (x - h)^2 + k$

The vertex is the point $(-2,-1)$.

5. The point $(-4,3)$ is symmetric to the y-intercept $(0,3)$ about the axis of symmetry, the vertical line $x = -2$.



25. $y = x^2 - 4$

26. $y = -x^2 + 9$

$\boxed{27.}$ $y = x^2 - 2x$

28. $y = 3x^2 + 2x$

29. $y = x^2 - 4x - 5$

30. $y = x^2 + 6x + 5$

31. $y = x^2 - x - 6$

32. $y = x^2 + 3x + 4$

33. $y = -x^2 - 2x + 3$

$\boxed{34.}$ $y = -x^2 + 7x - 6$

35. $y = x^2 - 4x + 4$

36. $y = x^2 + 2x + 1$

37. $y = 2x^2 - x - 3$

38. $y = 3x^2 + 4x + 1$

39. $y = -2x^2 + 3x - 1$

40. $y = -4x^2 + 4x + 3$

41. $y = x^2 + 2x + 5$

42. $y = x^2 - 3x + 6$

43. $y = 2x^2 + 4x + 5$

44. $y = 3x^2 + 6x + 4$

45. $y = -2x^2 + 4x + 5$

46. $y = -4x^2 + 12x - 9$

47. $y = -x^2 - 2x - 1$

48. $y = x^2 - 6x + 4$

See example 9–1 D–1.

49. The equation $s = 32t - 16t^2$ determines the distance s in feet that an object thrown vertically upward is above the ground in time t seconds. Find the time at which the object reaches its greatest height. Sketch the graph of the equation for $0 \leq t \leq 4$. (*Hint:* The greatest height is at the vertex.) When will the object hit the ground again? (*Hint:* When $s = 0$)

50. Given the equation $s = 64t - 8t^2$, do as instructed in exercise 49.

51. An arrow is shot vertically into the air with an initial velocity of 96 feet per second. If the height h in feet of the arrow at any time t seconds is given by

$h = 96t - 16t^2$

find the maximum height that the arrow will attain. When will the arrow come back to the ground?

52. A company's profit P when producing x units of a commodity in a given week is given by

$P = -x^2 + 100x - 1{,}000$

How many units must be produced to attain maximum profit?

53. Helen Nance owns a dress shop. She finds the profits from the shop are approximately given by

$P = -x^2 + 16x + 42$

where P is the profit when x dresses are sold daily. How many dresses must Helen sell daily to produce the maximum profit?

54. Russell Sanderson owns a pizza shop. From past results, he determines that the cost C of running the shop is given by the equation

$C = 3n^2 - 36n + 140$

where n is the number of pizzas sold daily. Find the number of pizzas Russell must sell to produce the lowest cost.

55. Tim Wesner sets up a Kool-Aid stand. He finds that the cost C of operating the stand is given by the equation

$C = 2x^2 - 20x + 100$

where x denotes the number of glasses of Kool-Aid sold. How many glasses of Kool-Aid should he sell for his cost to be the lowest?

56. The velocity distribution of natural gas flowing smoothly in a pipeline is given by

$V = 6x - x^2$

where V is the velocity in meters per second and x is the distance in meters from the inside wall of the pipe. What is the maximum velocity of the gas? (*Hint:* We want the second component V of the vertex of the graph of the equation.)

57. The power output P of an automobile alternator that generates 14 volts and has an internal resistance of 0.20 ohms is given by

$P = 14I - 0.20I^2$

At what current I does the generator generate maximum power and what is the maximum power?

See example 9–1 D–2.

58. A farmer wishes to fence in a rectangular piece of ground along a river bank for grazing his cattle. If he has 400 feet of fencing, what should be the dimensions of the rectangle to fence in the greatest area, using the river as one side of the rectangle?

59. Find the two numbers whose sum is 56 and whose product is the greatest possible value. (*Hint:* Let x be one number and $56 - x$ be the other number. Maximize their product.)

60. The sum of the length and the width of a rectangle is 48 inches. Find the length of the rectangle that would yield the greatest area. (*Hint:* Let $x =$ the length and $48 - x =$ the width. Use the formula $A = \ell w$.)

61. What are the dimensions of the largest rectangular plot of ground that can be enclosed by 42 meters of fence? (*Hint:* $2\ell + 2w = 42$. Solve for either ℓ or w and substitute in the formula $A = \ell w$. Maximize A.)

62. Find the maximum area of a rectangular plot of ground that can be enclosed with 28 rods of fencing.

63. Given that the difference between two numbers is 8, what is the minimum product of the two numbers? (*Hint:* Let x be one number and $x - 8$ be the other number.)

64. Find the minimum product of two numbers whose difference is 10.

Review exercises

Factor the following expressions. See sections 3–5 and 3–6.

1. $9y^2 - 6y + 1$ **2.** $4x^2 + 4x - 3$ **3.** $6y^2 - 24x^2$

Perform the indicated operations on rational expressions. See sections 4–2 and 4–3.

4. $\dfrac{4y}{y^2 - 25} - \dfrac{y}{y + 5}$ **5.** $\dfrac{16x^2}{7y} \div \dfrac{4x}{21y^2}$

6. Simplify the complex rational expression $\dfrac{\dfrac{x}{x + 4}}{\dfrac{x}{x - 3}}$. See section 4–4.

9–2 ■ More about parabolas

If we interchange the variables of the quadratic equation $y = ax^2 + bx + c$, $a \neq 0$, we obtain another quadratic equation of the form

$$x = ay^2 + by + c \ (a \neq 0)$$

The graph of this equation is also a parabola. However, these parabolas

1. open right when $a > 0$ (positive),
2. open left when $a < 0$ (negative).

The vertex will be the point with the least, or greatest, value of x.

We graph this equation in the same way, using the vertex and the intercepts. In this case, to find the coordinates of the vertex, we must write the equation in the form

$$x = a(y - k)^2 + h$$

and the point (h,k) is the vertex of the parabola. The equation of the axis of symmetry is $y = k$.

■ Example 9–2 A

1. Determine the vertex, the x- and y-intercepts, and graph the equation $x = y^2 + 4y - 5$.

a. Since $a = 1$ (positive), the graph opens right.

b. The x-intercept is $(-5,0)$ since $c = -5$.

c. Let $x = 0$.

$(0) = y^2 + 4y - 5$ Replace x with 0
$0 = (y + 5)(y - 1)$ Factor the right member
$y + 5 = 0$ or $y - 1 + 0$ Set each factor equal to 0
$y = -5$ $\quad y = 1$

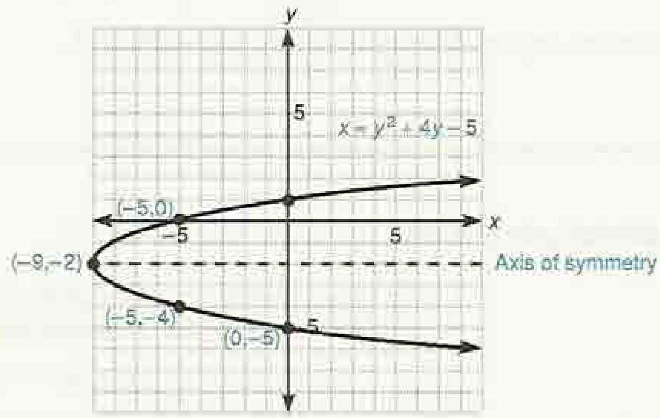
The y-intercepts are the points $(0,-5)$ and $(0,1)$.

d. Given $x = y^2 + 4y - 5$

$$x = (y^2 + 4y \quad\;) - 5 - (\;\;)$$
$$x = (y^2 + 4y + 4) - 5 - (4) \quad \text{Complete the square}$$
$$x = (y + 2)^2 - 9 \quad \text{Write as the square of a binomial}$$
$$x = [y - (-2)]^2 + (-9) \quad \text{Write in form } x = a(y - k)^2 + h$$

The vertex is the point $(-9,-2)$.

e. The point $(-5,-4)$ is symmetric to the x-intercept $(-5,0)$ about the axis of symmetry, $y = -2$.



2. Graph the quadratic equation $x = -y^2 + 3y + 4$ using completing the square.

a. The graph opens left since $a < 0$.

b. Since $c = 4$, the x-intercept is the point $(4,0)$.

c. Let $x = 0$.

$$(0) = -y^2 + 3y + 4 \quad \text{Replace } x \text{ with } 0$$
$$0 = y^2 - 3y - 4 \quad \text{Multiply by } -1$$
$$0 = (y - 4)(y + 1) \quad \text{Factor the right member}$$
$$y - 4 = 0 \text{ or } y + 1 = 0 \quad \text{Set each factor equal to } 0$$
$$y = 4 \qquad y = -1$$

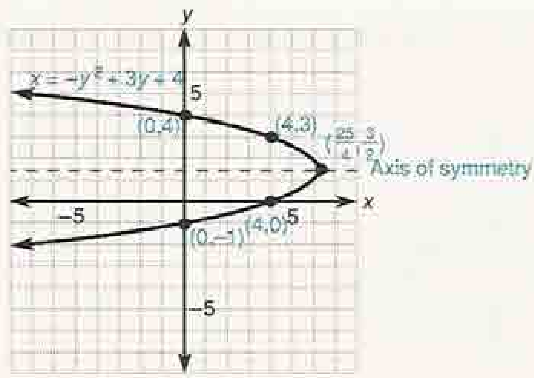
The y-intercepts are the points $(0,4)$ and $(0,-1)$.

d. Given $x = -y^2 + 3y + 4$

$$= -1(y^2 - 3y + \quad\;) + 4 - (\;\;)$$
$$= -1\left(y^2 - 3y + \frac{9}{4}\right) + 4 - \left(-\frac{9}{4}\right) \quad \text{Complete the square}$$
$$= -1\left(y - \frac{3}{2}\right)^2 + \frac{25}{4} \quad \text{Write in form } x = a(y - k)^2 + h$$

The vertex is the point $\left(\frac{25}{4},\frac{3}{2}\right)$.

e. The point $(4,3)$ is symmetric to the x-intercept $(4,0)$ about the axis of symmetry, $y = \frac{3}{2}$.



▶ ***Quick check*** Graph the equation $x = -y^2 - 2y + 3$. ■

Forms of the equation of a parabola

Given the parabola opens right when $a > 0$ and opens left when $a < 0$,

1. The *general form* of the equation of the parabola is
$$x = ay^2 + by + c$$
2. The *standard form* of the equation of the parabola is
$$x = a(y - k)^2 + h$$

Mastery points

Can you

- Graph a parabola of the form $x = ay^2 + by + c$?

Exercise 9–2

Graph the following parabolas. See example 9–2 A.

Example $x = -y^2 - 2y + 3$

Solution 1. Since $a = -1$ (negative), the graph opens to the left.
2. Since $c = 3$, the x-intercept is the point (3,0).
3. Let $x = 0$.

$(0) = -y^2 - 2y + 3$	Replace x with 0
$0 = y^2 + 2y - 3$	Multiply each member by -1
$0 = (y + 3)(y - 1)$	Factor the right member
$y + 3 = 0$ or $y - 1 = 0$	Set each factor equal to 0
$y = -3 \qquad y = 1$	Solve for y

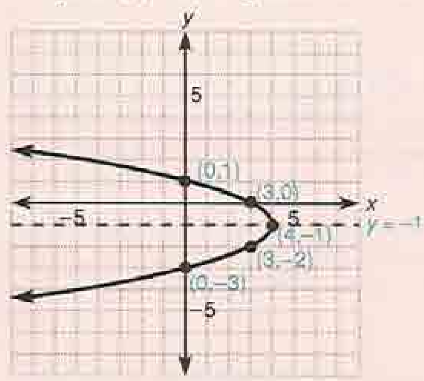
The y-intercepts are the points $(0,-3)$ and $(0,1)$.

4. Given $x = -y^2 - 2y + 3$

$x = -1(y^2 + 2y + \quad) + 3 - (\quad)$	Factor -1 from $-y^2 - 2y$
$x = -1(y^2 + 2y + 1) + 3 - (-1)$	Complete the square
$x = -1(y + 1)^2 + 4$	Write as the square of a binomial
$x = -1[y - (-1)]^2 + 4$	Write in form $x = a(y - k)^2 + h$

The vertex is the point $(4,-1)$.

5. The point $(3,-2)$ is symmetric to the x-intercept, $(3,0)$ about the axis of symmetry, $y=-1$.



1. $x=3(y-1)^2+2$
2. $x=2(y-3)^2+1$
3. $x=-2(y+3)^2-2$
4. $x=-1(y+2)^2-1$
5. $x=y^2-3y+2$
6. $x=y^2-2y-8$
7. $x=-y^2+2y+3$
8. $x=3y^2-y-6$
9. $x=5y^2+2y-3$
10. $x=-2y^2+5y-3$
11. $x=y^2-2$
12. $x=-y^2+4$
13. $x=y^2$
14. $x=-y^2$
15. $x=-5y^2$
16. $x=-9y^2$
17. $x=y^2-y$
18. $x=-y^2+6y$
19. $x=-2y^2+3y$

Review exercises

1. Solve the system of linear equations $4x-y=2$, $x+2y=5$. See section 8–1.

Simplify the following expressions. See section 3–3.

2. $(2a^{-2}b^3c^0)^{-3}$
3. $\left[\dfrac{8x^{-5}}{16x^3}\right]^3$

Find the x- and y-intercepts of the following equations. See section 7–1.

4. $x+2y=4$
5. $4x+4y=12$

Find the distance between the following sets of points. See section 7–2.

6. $(-3,2)$ and $(1,7)$
7. $(6,-5)$ and (x,y)
8. (x,y) and (h,k)

9–3 ■ The circle

In sections 9–1 and 9–2, we discussed second-degree equations having just one second-degree term. Now we will consider second-degree equations that contain two second-degree terms, x^2 and y^2. The circle is one of the conic sections that contains these terms in the equation.

Definition of a circle

A **circle** is defined to be the set of all points in a plane that are at a fixed distance from a fixed point.

The fixed point is called the *center* of the circle, denoted by C, and the fixed distance from the center to all points on the circle is called the *radius* of the circle, denoted by r. See figure 9–4.

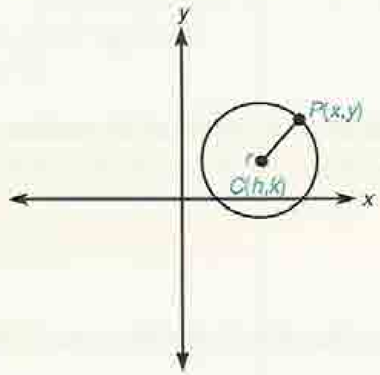


Figure 9–4

Recall the distance formula we discussed in chapter 7.

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

If we square each member of this equation, we have

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

Replace the point (x_2,y_2) with the arbitrary point on the circle $P(x,y)$, the point (x_1,y_1) with the center $C(h,k)$, and the distance d with the radius r to obtain the *standard form* of the equation of the circle in figure 9–4.

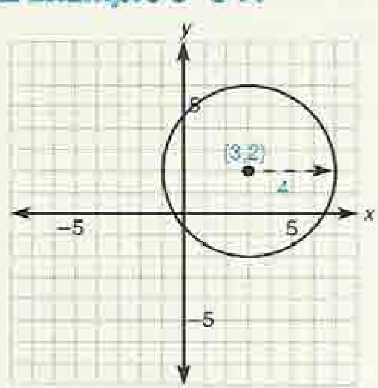
Equation of the circle

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h,k) is the center of the circle and r is the length of the radius.

We call this the *standard form* of the equation of a circle.

Example 9–3 A



Find the equation for the circle with center at $C(3,2)$ and radius $r = 4$ units.

We are given $h = 3$, $k = 2$, and $r = 4$.

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Standard form}$$
$$[x - (3)]^2 + [y - (2)]^2 = (4)^2 \quad \text{Replace } h \text{ with 3, } k \text{ with 2, and } r \text{ with 4}$$
$$(x - 3)^2 + (y - 2)^2 = 16$$

The equation of the circle is $(x - 3)^2 + (y - 2)^2 = 16$.

▶ ***Quick check*** Find the equation for the circle with center at $C(4,-3)$ and radius $r = 5$ units.

A special case exists when the center is at the origin. Then

$$(h,k) = (0,0)$$

and the equation of the circle is given by

$$(x - 0)^2 + (y - 0)^2 = r^2$$
$$x^2 + y^2 = r^2$$

Equation of a circle with center at origin

The equation of a circle with the center at the origin, $C(0,0)$, and radius r is

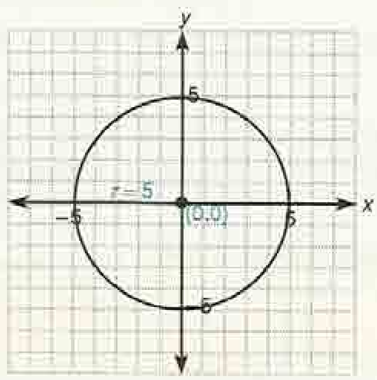
$$x^2 + y^2 = r^2.$$

■ Example 9–3 B

Find the equation of a circle with center $C(0,0)$ and radius $r = 5$. Graph the circle.

Using $x^2 + y^2 = r^2$, we substitute to obtain

$x^2 + y^2 = 5^2$ Replace r with 5
$x^2 + y^2 = 25$ $5^2 = 25$



■

Often the equation of a circle is stated in the *general form*

$$ax^2 + ay^2 + bx + cy + d = 0$$

The following examples illustrate the procedure for finding the coordinates of the center and finding the length of the radius of a given circle.

■ Example 9–3 C

Determine the coordinates of the center and the length of the radius of the given circle. Sketch the graph.

1. $x^2 + y^2 - 10x - 8y - 23 = 0$
   We first rewrite this equation in the form $(x - h)^2 + (y - h)^2 = r^2$ by completing the square in both x and y.

$$(x^2 - 10x) + (y^2 - 8y) - 23 = 0$$ Group terms having same variables

$$(x^2 - 10x + \quad) + (y^2 - 8y + \quad) = 23 + (\quad) + (\quad)$$ Add 23 to each member

Completing the square in both x and y (adding same quantities to both members), we have

Add 25 to each member

$$(x^2 - 10x + 25) + (y^2 - 8y + 16) = 23 + 25 + 16$$

Add 16 to each member

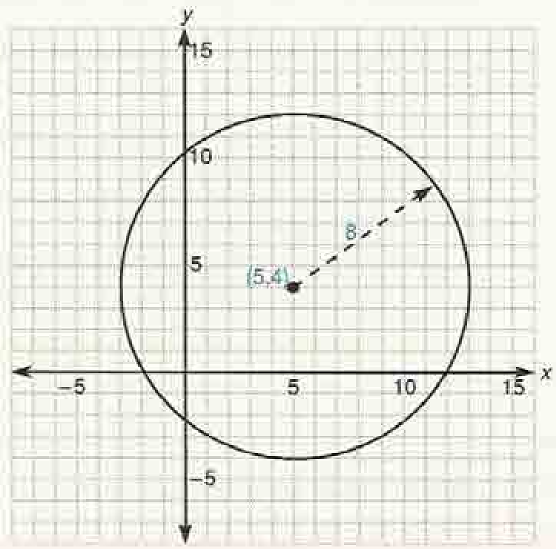
Complete the square for each quadratic expression

$$(x - 5)^2 + (y - 4)^2 = 64$$

$$(x - 5)^2 + (y - 4)^2 = 8^2$$

Write as the square of a binomial; combine

The circle has center $C(5,4)$ and radius $r = 8$.
Plot the center $(5,4)$ and draw a circle that is 8 units in each direction from this point.



2. $3x^2 + 3y^2 - 18x + 12y + 27 = 0$
To complete the square, we must make the coefficients of the squared terms 1. To accomplish this, divide *all* terms by 3 to obtain

$$x^2 + y^2 - 6x + 4y + 9 = 0$$ Divide each term by 3

$$(x^2 - 6x) + (y^2 + 4y) + 9 = 0$$ Group terms having same variable

$$(x^2 - 6x) + (y^2 + 4y) = -9$$ Add -9 to each member

Completing the square and adding the same quantities to each member of the equation,

Add 9 to each member

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$

Add 4 to each member

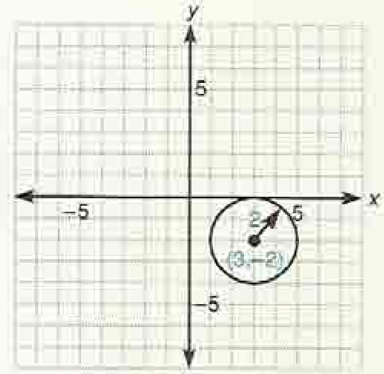
Complete the square for each quadratic expression

$$(x - 3)^2 + (y + 2)^2 = 4$$ Write as the square of a binomial; combine

$$(x - 3)^2 + (y + 2)^2 = 2^2$$

$$(x - 3)^2 + [y - (-2)]^2 = 2^2$$ Write in standard form

Therefore the circle has center $C(3,-2)$ and radius $r = 2$.
Plot the center $(3,-2)$ and draw a circle that is 2 units in each direction from this point.



▶ ***Quick check*** Determine the coordinates of the center and the length of the radius for $x^2 + y^2 + 2x - 4y - 4 = 0$. ■

Mastery points

Can you

- Write the equation of a circle given the coordinates of the center and the radius?
- Find the coordinates of the center and the length of the radius of a circle given the equation?
- Given the general form of the equation of a circle, find the standard form of the equation of the circle?
- Graph the equation of a circle?

Exercise 9–3

Write the equation for each of the following circles. See example 9–3 A.

Example Center at $C(4,-3)$ and radius $r = 5$ units

Solution We are given $h = 4$, $k = -3$, and $r = 5$.
Using $(x - h)^2 + (y - k)^2 = r^2$,

$$[x - (4)]^2 + [(y - (-3)]^2 = (5)^2 \qquad \text{Replace } h \text{ with } 4, k \text{ with } -3, \text{ and } r \text{ with } 5$$
$$(x - 4)^2 + (y + 3)^2 = 25$$

1. Center $C(1,2)$ and radius 2
2. Center $C(-5,7)$ and radius 4
3. Center $C(4,-3)$ and radius $\sqrt{6}$
4. Center $C(-4,-5)$ and radius $2\sqrt{2}$
5. Center at the origin and radius 3
6. Center at the origin and radius $\sqrt{5}$

Write the equation in the form $x^2 + y^2 + Bx + Cy + D = 0$, where B, C, and D are integers.

7. Center $C(-5,2)$ and radius 1
8. Center $C(1,-3)$ and radius $\sqrt{10}$
9. Center at the origin and radius 6
10. Center $C(0,7)$ and radius $\sqrt{7}$

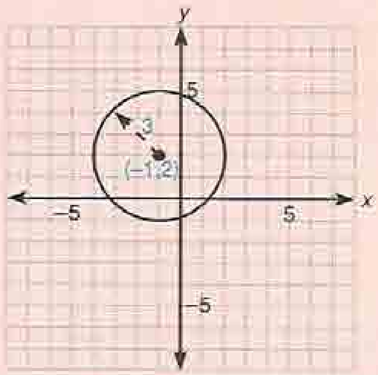
Determine the coordinates of the center and the radius of each of the following circles. See example 9–3 C.

Example $x^2 + y^2 + 2x - 4y - 4 = 0$

Solution We must write the equation in standard form $(x - h)^2 + (y - k)^2 = r^2$.

$$(x^2 + 2x) + (y^2 - 4y) - 4 = 0 \quad \text{Group terms with same variables}$$
$$(x^2 + 2x \quad) + (y^2 - 4y \quad) = 4 + (\quad) + (\quad) \quad \text{Add 4 to each member}$$
$$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4 \quad \text{Add 1 and 4 to each member to complete the square}$$
$$(x + 1)^2 + (y - 2)^2 = 9 \quad \text{Write as binomial squares and add}$$
$$[x - (-1)]^2 + (y - 2)^2 = 3^2 \quad \text{Write in form } (x - h)^2 + (y - k)^2 = r^2$$

The center is at $C(-1,2)$ and the radius is 3 units.



11. $(x - 3)^2 + (y - 2)^2 = 49$
12. $(x - 1)^2 + (y - 4)^2 = 16$
13. $(x - 5)^2 + (y + 3)^2 = 8$
14. $(x + 1)^2 + (y - 7)^2 = 11$
15. $(x + 1)^2 + (y + 9)^2 = 25$
16. $(x + 6)^2 + (y + 2)^2 = 24$
17. $x^2 + y^2 = 36$
18. $x^2 + y^2 = 1$
19. $x^2 + y^2 = 2$
20. $x^2 + y^2 = 18$
21. $x^2 + y^2 + 4x - 6y - 23 = 0$
22. $x^2 + y^2 + 4x - 4y - 8 = 0$
23. $x^2 + y^2 + 6x - 8y = 0$
24. $x^2 + y^2 - 16x + 30y = 0$
25. $2x^2 + 2y^2 + 8x - 12y = 74$
26. $3x^2 + 3y^2 - 24x + 12y - 87 = 0$
27. $3x^2 + 3y^2 + 18x - 6y = 45$
28. $2x^2 + 2y^2 - 4x + 4y = 22$

Graph each of the following circles. See examples 9–3 A and B.

29. $x^2 + y^2 = 1$
30. $x^2 + y^2 = 36$
31. $3x^2 + 3y^2 = 12$
32. $4x^2 + 4y^2 = 24$
33. $(x - 4)^2 + (y + 3)^2 = 4$
34. $(x + 2)^2 + (y - 5)^2 = 5$
35. $x^2 + y^2 - 2x = 15$
36. $x^2 + y^2 - 4x = 0$
37. $x^2 + y^2 - 2x + 4y - 20 = 0$
38. $x^2 + y^2 + 6x - 2y - 39 = 0$
39. $2x^2 + 2y^2 - 12x + 4y = -12$
40. $4x^2 + 4y^2 - 16x + 24y = 92$
41. Using the point (h,k), any point on the circle (x,y), r for the radius, and the distance formula, derive the equation of the circle $(x - h)^2 + (y - k)^2 = r^2$.
42. Find the equation of the circle passing through the origin with center $C(0,5)$.
43. Find the equation of the circle passing through the origin with center $C(-2,0)$.
44. Find the equation of the circle passing through the origin with center $C(3,-2)$.

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Identify the figure and graph the following equations.

45. $y = x^2 - x - 2$

46. $3x - 2y = -6$

47. $3x^2 = 12 - 3y^2$

48. $x - y^2 = 4x + 3$

Review exercises

1. Evaluate $\begin{vmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 2 & 0 & -4 \end{vmatrix}$. See section 8–4.

2. Graph the equation $3x - y = 6$. See section 7–1.

3. Combine $\sqrt{8xy^3} - \sqrt{128xy^3}$ and simplify. See section 5–5.

4. Given $P(x) = 4x^2 - 2x + 8$, find $P(-1)$. See section 1–5.

5. Given the line $2x - 3y = -6$, find the equation of the line through $(2,-1)$ that is perpendicular to the given line. See section 7–3.

9–4 ■ The ellipse and the hyperbola

The ellipse

Two other conics containing x^2 and y^2 terms are the ellipse and the hyperbola. In this book, we shall consider only ellipses and hyperbolas with the center at the origin, (0,0).

Definition

An **ellipse** is the set of all points in a plane such that the sum of the distances from each point on the ellipse to two fixed points in the plane is a positive constant, k.

In figure 9–5, for every point on the ellipse, $d_1 + d_2 = k$ (constant). We call the fixed points the *foci* of the ellipse. Each fixed point is a **focus** of the ellipse, which we denote by F_1 and F_2. See figure 9–5.

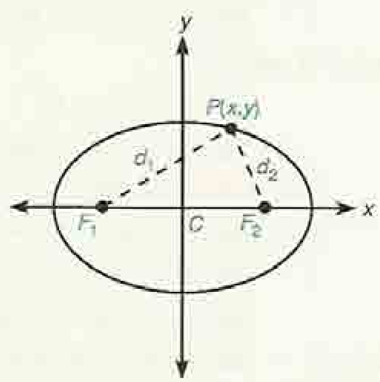


Figure 9–5

Given the x-intercepts of the ellipse at $(a,0)$ and $(-a,0)$ and the y-intercepts at $(0,b)$ and $(0,-b)$ (a and b are both positive), it can be shown that the equation of the ellipse with its center at the origin $(0,0)$ is given as follows:

Equation of the ellipse

The **standard form** of the equation of the ellipse with the center at the origin is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; a \neq b$$

where the intercepts are $(a,0)$, $(-a,0)$, $(0,b)$, and $(0,-b)$.

To graph an ellipse with its center at $(0,0)$, plot the four intercepts $(a,0)$, $(-a,0)$, $(0,b)$, $(0,-b)$ and sketch the ellipse through the intercepts. See figure 9–6.

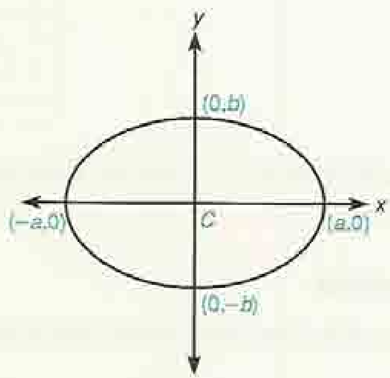


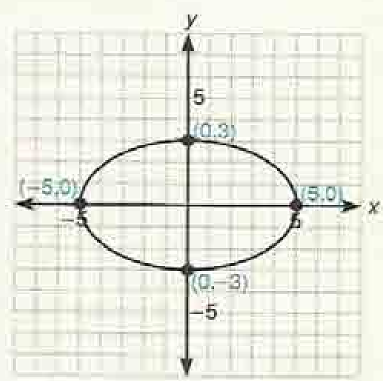
Figure 9–6

Example 9–4 A

Find the coordinates of the intercepts of the following ellipses and sketch the graph.

1. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

a. $a^2 = 25$, so $a = 5$.
b. $b^2 = 9$, so $b = 3$.



The x-intercepts are $(-5,0)$ and $(5,0)$ while the y-intercepts are $(0,-3)$ and $(0,3)$.

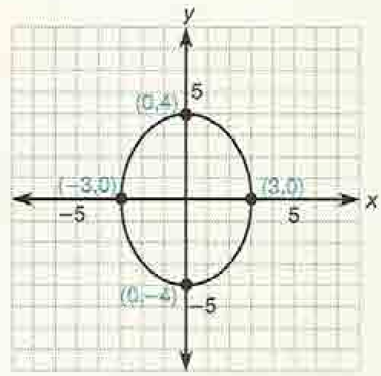
2. $16x^2 + 9y^2 = 144$

To obtain the standard form of the equation, we must divide each member by 144, the constant in the right member, since that constant must be 1.

$$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144} \qquad \text{Divide each term by 144}$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \qquad \text{Reduce in each term}$$

a. $a^2 = 9$, so $a = 3$.
b. $b^2 = 16$, so $b = 4$.
The x-intercepts are $(-3,0)$ and $(3,0)$ while the y-intercepts are $(0,-4)$ and $(0,4)$.



▶ ***Quick check*** Find the coordinates of the intercepts of $25x^2 + 4y^2 = 100$. Sketch the graph.

The hyperbola

The equation of the last conic section, the hyperbola, is similar to the equation of the ellipse.

Definition of a hyperbola

A **hyperbola** is the set of all points in the plane such that the absolute value of the *difference* between the distances from each point on the hyperbola to two fixed points is a constant, *k*.

The fixed points are again called the **foci** of the hyperbola. See figure 9–7.

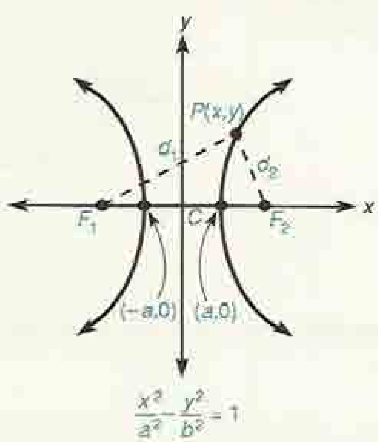


Figure 9–7

In figure 9–7, for every point on the hyperbola, $|d_1 - d_2| = k$ (constant). Using the above relationship, we can show (but will not do so here) that the standard form of the equation of a hyperbola with center C at the origin is as follows:

Equations of the hyperbola

The **standard form** of the equation of the hyperbola with the center at the origin is given by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

with x-intercepts a and $-a$ and no y-intercepts (see figure 9–7) or

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

with y-intercepts b and $-b$ and no x-intercepts (see figure 9–8).

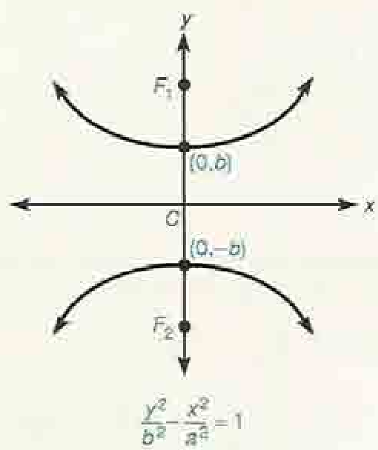


Figure 9–8

Solving the equations $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ for y in terms of a, b, and x, we can show that the graph of each of these hyperbolas will approach (get closer and closer to) but not cross the lines whose equations are

$$y = \frac{b}{a}x \quad \text{and} \quad y = -\frac{b}{a}x$$

We call such lines **asymptotes.** They are indicated by dashed lines. These lines are useful for sketching the graph of the hyperbola and are found in the following way:

Asymptotes of a hyperbola

The asymptotes of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ are the extended diagonals of the rectangle whose vertices are (a,b), $(a,-b)$, $(-a,b)$, and $(-a,-b)$.

To illustrate, given the equation

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

then, (1) $a^2 = 4$, so $a = 2$

(2) $b^2 = 25$, so $b = 5$.

We can now construct a rectangle with corners (2,5), (2,−5), (−2,5), and (−2,−5). To obtain the asymptotes for this hyperbola, draw the diagonals. See figure 9–9.

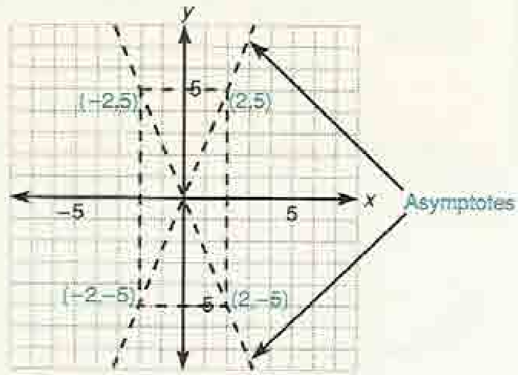


Figure 9–9

Using the intercepts and the asymptotes, we can sketch the graph of a hyperbola with its center at the origin (0,0). Since $a = 2$, $b = 5$, and the equations of the asymptotes are $y = \pm\frac{b}{a}x$, then

$$y = -\frac{b}{a}x = -\frac{5}{2}x \quad \text{and} \quad y = \frac{b}{a}x = \frac{5}{2}x$$

when we replace a with 2 and b with 5. The graph of the equation is shown in figure 9–10.

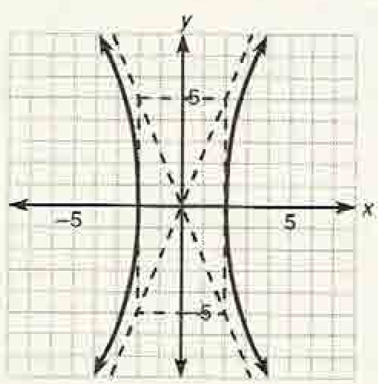


Figure 9–10

In summary, to graph a hyperbola of either of the two standard forms, we follow this procedure:

To graph a hyperbola

Step 1 Write the equation in standard form.

Step 2 Locate the intercepts at $(-a,0)$ and $(a,0)$ if the coefficient of x^2 is positive and at $(0,-b)$ and $(0,b)$ if the coefficient of y^2 is positive.

Step 3 Draw the rectangle (in dashed lines) with vertices at (a,b), $(a,-b)$, $(-a,b)$, and $(-a,-b)$.

Step 4 Sketch the asymptotes (in dashed lines) that are extensions of the diagonals of the rectangle.

Step 5 Sketch the hyperbola through the intercepts and approach the lines that are the asymptotes.

Example 9–4 B

Determine the intercepts of the following hyperbolas. Sketch the graphs.

1. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

   a. $a^2 = 16$, so $a = 4$.

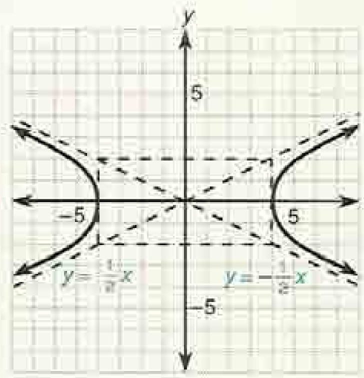
   b. $b^2 = 4$, so $b = 2$.

   The x-intercepts are $(-4,0)$ and $(4,0)$ and there are no y-intercepts. The rectangle is formed by the points $(4,2)$, $(4,-2)$, $(-4,2)$, and $(-4,-2)$.

   The equations of the asymptotes are

   $y = \dfrac{2}{4}x = \dfrac{1}{2}x$ and

   $y = -\dfrac{2}{4}x = -\dfrac{1}{2}x$.



2. $9y^2 - 4x^2 = 36$

   To get the equation into one of the standard forms, divide each term by 36. Then

   $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$

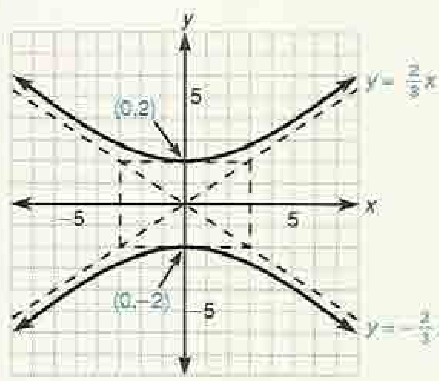
   a. $b^2 = 4$, so $b = 2$.

   b. $a^2 = 9$, so $a = 3$.

   The y-intercepts are $(0,-2)$ and $(0,2)$; there are no x-intercepts. The rectangle is formed by the points $(3,2)$, $(3,-2)$, $(-3,2)$, and $(-3,-2)$.

   The equations of the asymptotes are

   $y = \dfrac{2}{3}x$ and $y = -\dfrac{2}{3}x$.



▶ ***Quick check*** Determine the intercepts and equations of the asymptotes of $9y^2 - 9x^2 = 81$. Sketch the graph. ■

We now summarize the characteristics of the conic sections.

Equation	Conic section	Characteristics
$y = ax^2 + bx + c$	Parabola	If $a < 0$, opens down If $a > 0$, opens up
$x = ay^2 + by + c$	Parabola	If $a > 0$, opens right If $a < 0$, opens left
$(x - h)^2 + (y - k)^2 = r^2$	Circle	Center at (h,k) and radius r
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a \neq b)$	Ellipse	x-intercepts, $(a,0)$ and $(-a,0)$ y-intercepts, $(0,b)$ and $(0,-b)$
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Hyperbola	x-intercepts, $(a,0)$ and $(-a,0)$. No y-intercepts
$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$	Hyperbola	y-intercepts, $(0,b)$ and $(0,-b)$. No x-intercepts

■ **Example 9–4 C**

Identify each equation as a parabola, a circle, an ellipse, or a hyperbola.

1. $x^2 - 2y = 0$
   Since the equation contains the second power of x and the first power of y, then y is linear and the graph is a **parabola.**
2. $4x^2 = 12 - 3y^2$
   Adding $3y^2$ to each member, we obtain the equation
   $$4x^2 + 3y^2 = 12 \quad \text{or} \quad \frac{x^2}{3} + \frac{y^2}{4} = 1$$
   The equation of an **ellipse.**
3. $3y^2 = 9 + 3x^2$
   Adding $-3x^2$ to each member, we obtain
   $$3y^2 - 3x^2 = 9 \quad \text{or} \quad \frac{y^2}{3} - \frac{x^2}{3} = 1$$
   The equation of a **hyperbola.** ■

Mastery points

Can you

- Find the coordinates of the x- and y-intercepts for an ellipse and a hyperbola?
- Sketch the graphs of ellipses and hyperbolas?
- Find the equations of the asymptotes of a hyperbola and graph them?
- Identify the equation of a parabola, a circle, an ellipse, and a hyperbola and sketch their graphs?

Exercise 9–4

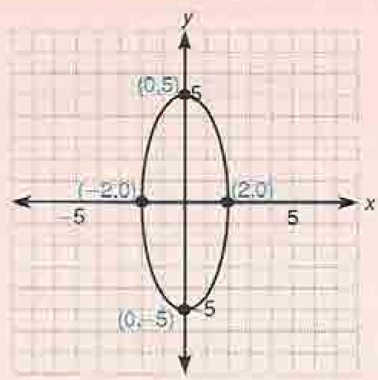
Find the x- and y-intercepts of each given ellipse. Sketch the graph of each equation. See example 9–4 A.

Example $25x^2 + 4y^2 = 100$

Solution To get the equation in standard form, divide each member by 100.

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

1. $a^2 = 4$, so $a = 2$.
2. $b^2 = 25$, so $b = 5$.



The x-intercepts are $(-2,0)$ and $(2,0)$, the y-intercepts are $(0,-5)$ and $(0,5)$.

1. $\frac{x^2}{9} + \frac{y^2}{25} = 1$
2. $\frac{x^2}{4} + \frac{y^2}{9} = 1$
3. $\frac{x^2}{49} + \frac{y^2}{25} = 1$
4. $\frac{x^2}{16} + \frac{y^2}{36} = 1$
5. $x^2 + \frac{y^2}{9} = 1$
6. $x^2 + \frac{y^2}{4} = 1$
7. $\frac{x^2}{16} + y^2 = 1$
8. $36x^2 + 9y^2 = 324$
9. $x^2 + 25y^2 = 100$
10. $16x^2 + y^2 = 64$
11. $3x^2 + 4y^2 = 12$
12. $9x^2 + 2y^2 = 18$
13. $8x^2 + y^2 = 16$
14. $x^2 + 3y^2 = 27$

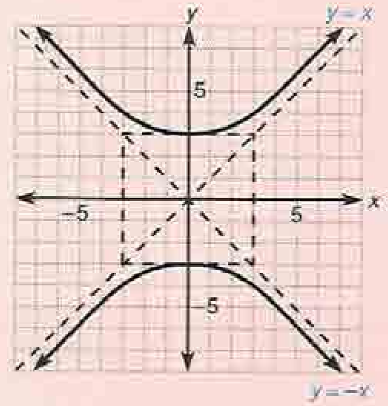
Find the x-intercepts, or y-intercepts, and the equations of the asymptotes of the hyperbola for the given equation. Sketch the graph of the equation. See example 9–4 B.

Example $9y^2 - 9x^2 = 81$

Solution Divide each member by 81 to get the equation in standard form.

$$\frac{9y^2}{81} - \frac{9x^2}{81} = \frac{81}{81}$$
$$\frac{y^2}{9} - \frac{x^2}{9} = 1$$

a. $b^2 = 9$, so $b = 3$.
b. $a^2 = 9$, so $a = 3$.
y-intercepts, $(0,-3)$, $(0,3)$; no x-intercepts
The rectangle has vertices $(3,3)$, $(3,-3)$, $(-3,3)$, and $(-3,-3)$.
The equations of the asymptotes are $y = -x$ and $y = x$.



15. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
16. $\frac{x^2}{4} - \frac{y^2}{25} = 1$
17. $\frac{y^2}{9} - \frac{x^2}{4} = 1$
18. $\frac{y^2}{16} - \frac{x^2}{25} = 1$
19. $y^2 - \frac{x^2}{9} = 1$
20. $y^2 - \frac{x^2}{4} = 1$
21. $x^2 - \frac{y^2}{16} = 1$
22. $9x^2 - y^2 = 36$
23. $y^2 - 25x^2 = 25$
24. $y^2 - 16x^2 = 64$
25. $25y^2 - 16x^2 = 400$
26. $16y^2 - 9x^2 = 144$
27. $25x^2 - 4y^2 = -100$
28. $2y^2 - 3x^2 = -18$
29. $2x^2 - 9y^2 = -36$
30. $6x^2 - 6y^2 = -1$

Write each of the following equations in standard form. Identify as a parabola, a circle, an ellipse, or a hyperbola. See example 9–4 C.

31. $x^2 = 9 - y^2$
32. $9x^2 = 36 + 4y^2$
33. $x^2 + 3 = y$
34. $2x^2 + y^2 = 8$
35. $4y^2 = 25 - 4x^2$
36. $y^2 - x^2 = 25$
37. $y^2 = 121 - x^2$
38. $8y^2 = 24 + 8x^2$
39. $x^2 + y = 8$
40. $18 - 2y^2 = 3x^2$

Review exercises

Perform the indicated operations on the following rational expressions. See sections 4–2 and 4–3.

1. $\frac{2y + 1}{y - 3} \cdot \frac{y^2 - 2y - 3}{2y^2 - 5y - 3}$
2. $\frac{4z}{3z + 2} - \frac{6z}{2z - 1}$
3. Simplify the complex rational expression $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}}$.

See section 4–4.

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Find the solution set of the following quadratic equations. See section 6–1.

4. $4x^2 = 20$

5. $5x^2 - 4x = 0$

Perform the indicated operations. See sections 5–5 and 5–6.

6. $(\sqrt{3} - 2\sqrt{2}) - (3\sqrt{3} + \sqrt{2})$

7. $(2\sqrt{5} + \sqrt{3})(3\sqrt{5} - \sqrt{3})$

9–5 ■ Systems of nonlinear equations

In section 8–1, we discussed the solution(s) of systems of two linear equations. Now we will consider systems of two equations in which at least one of the equations is quadratic. These are called *systems of nonlinear equations*. The methods used to solve systems of linear equations can be used to solve systems involving quadratic equations. The following examples illustrate this kind of system.

■ Example 9–5 A

Find the solution set of the given system of equations.

$$y = x^2 - 2x + 1 \quad (1)$$
$$x + y = 3 \quad (2)$$

We first solve equation (2) for y to obtain the equivalent system.

$$y = x^2 - 2x + 1$$
$$y = 3 - x$$

Substitute $3 - x$ for y in equation (1) and solve for x.

$$(3 - x) = x^2 - 2x + 1 \quad \text{Replace } y \text{ with } 3 - x$$
$$0 = x^2 - x - 2 \quad \text{Add } x \text{ and } -3 \text{ to each member}$$
$$x^2 - x - 2 = 0 \quad \text{Write equation in standard form}$$
$$(x - 2)(x + 1) = 0 \quad \text{Factor the left member}$$
$$x - 2 = 0 \quad \text{or} \quad x + 1 = 0 \quad \text{Set each factor equal to 0}$$
$$x = 2 \qquad\qquad x = -1$$

Using equation (2), substitute for x and solve for y.

(1) Let $x = 2$.

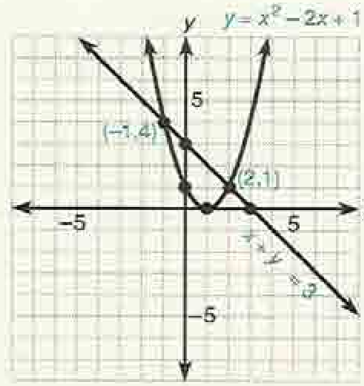
$$x + y = 3$$
$$(2) + y = 3 \quad \text{Replace } x \text{ with 2}$$
$$y = 1$$

(2) Let $x = -1$.

$$x + y = 3$$
$$(-1) + y = 3 \quad \text{Replace } x \text{ with } -1$$
$$y = 4$$

The solutions are the ordered pairs (2,1) and (−1,4). The solution set is $\{(2,1), (-1,4)\}$.

Equation (1) is a parabola and equation (2) is a straight line. Graphing equation (1) using the vertex and the intercepts and equation (2) using the intercepts, we see that the two graphs intersect at the points (−1,4) and (2,1).



▶ ***Quick check*** Find the solution set of the system $x^2 + y^2 = 9$
$2x - y = 3.$ ■

When both equations making up the system are quadratic equations, we use the method of solution by **elimination.** The result is then a quadratic equation in one variable, and we proceed as in our preceding example.

■ ***Example 9–5 B***

Find the solution set of the system.

$x^2 + y^2 = 25 \quad (1)$
$x^2 + 4y^2 = 52 \quad (2)$

To eliminate the variable x, we multiply equation (1) by -1 and add to equation (2).

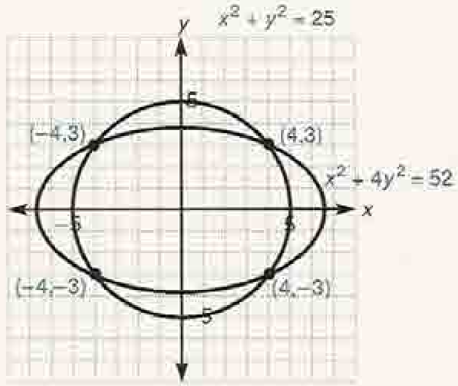
$$\begin{array}{lcrl} x^2 + y^2 = 25 & \text{Multiply by } -1 \rightarrow & -x^2 - y^2 = -25 & \\ x^2 + 4y^2 = 52 & & \underline{x^2 + 4y^2 = 52} & \\ & & 3y^2 = 27 & \text{Add members} \\ & & y^2 = 9 & \text{Divide by 3} \\ & & y = \pm 3 & \text{Extract the roots} \end{array}$$

Substituting 3 and -3 for y in either original equation, say $x^2 + y^2 = 25$, we obtain corresponding values of x.

$$\begin{array}{rrl} \text{When } y = 3, & \text{When } y = -3, & \\ x^2 + (3)^2 = 25 & x^2 + (-3)^2 = 25 & \text{Replace } y \text{ with } 3, -3 \\ x^2 + 9 = 25 & x^2 + 9 = 25 & \text{Square as indicated} \\ x^2 = 16 & x^2 = 16 & \text{Subtract 9 from each member} \\ x = \pm 4 & x = \pm 4 & \text{Extract the roots} \end{array}$$

Thus (a) $y = 3, x = 4$; (b) $y = 3, x = -4$; (c) $y = -3, x = 4$; and (d) $y = -3, x = -4$. The solution set is $\{(4,3), (-4,3), (4,-3), (-4,-3)\}$.

Equation (1) is a circle and equation (2) is an ellipse with centers at the origin, (0,0). If we graph both equations using their x- and y-intercepts, we see the graphs intersect at the points $(-4,3)$, $(-4,-3)$, $(4,3)$, and $(4,-3)$.



▶ ***Quick check*** Find the solution set of the system $x^2 + y^2 = 18$
$2x^2 - y^2 = -6$.

Mastery points

Can you

- Find the solution set of a system of equations that contain one linear and one quadratic equation?
- Find the solution set of a system of equations that contain two quadratic equations?

Exercise 9–5

Find the solution set of each system of equations by substitution. Sketch the graphs of the systems in exercises 1–6. See example 9–5 A.

Example $x^2 + y^2 = 9$ (1)
$2x - y = 3$ (2)

Solution We first solve equation (2) for y.

$$2x - y = 3$$
$$-y = -2x + 3 \quad \text{Subtract } 2x \text{ from each member}$$
$$y = 2x - 3 \quad \text{Multiply each member by } -1$$

We now substitute $2x - 3$ for y in equation (1).

$$x^2 + y^2 = 9$$
$$x^2 + (2x - 3)^2 = 9 \quad \text{Replace } y \text{ with } 2x - 3$$
$$x^2 + 4x^2 - 12x + 9 = 9 \quad \text{Square in left member}$$
$$5x^2 - 12x = 0 \quad \text{Combine like terms and subtract 9 from each member}$$
$$x(5x - 12) = 0 \quad \text{Factor in left member}$$
$$x = 0 \quad \text{or} \quad 5x - 12 = 0 \quad \text{Set each factor equal to 0}$$
$$x = 0 \quad \text{or} \quad x = \frac{12}{5}$$

Substitute 0 and $\frac{12}{5}$ for x in equation (2) and solve for y.

Let $x = 0$. Let $x = \frac{12}{5}$.

$$2(0) - y = 3 \qquad 2\left(\frac{12}{5}\right) - y = 0 \qquad \text{Replace } x \text{ with } 0, \frac{12}{5}$$

$$-y = 3 \qquad \frac{24}{5} - y = 3$$

$$y = -3 \qquad -y = 3 - \frac{24}{5}$$

$$-y = -\frac{9}{5}$$

$$y = \frac{9}{5}$$

The solutions are $(0, -3)$ and $\left(\frac{12}{5}, \frac{9}{5}\right)$.

The solution set is $\left\{(0, -3), \left(\frac{12}{5}, \frac{9}{5}\right)\right\}$.

1. $x^2 + y^2 = 5$
   $x + y = 1$
2. $x^2 + y^2 = 1$
   $x + 2y = 2$
3. $x^2 - y^2 = 24$
   $x + 2y = -3$
4. $x^2 - y^2 = 9$
   $x + y = 5$
5. $y = x^2 - 2x + 1$
   $y = 3 - x$
6. $y = x^2 - 4x + 4$
   $x + y = 2$
7. $x^2 + 2y^2 = 12$
   $2x = y + 2$
8. $x^2 + 3y^2 = 3$
   $x - 3y = 0$
9. $x^2 - y^2 = 15$
   $xy = 4$
10. $x^2 - y^2 = 35$
    $xy = 6$
11. $x^2 + y^2 = 8$
    $xy = 4$
12. $3x^2 - 4y^2 = 12$
    $x = 4$
13. $2x^2 + y^2 = 4$
    $y = -1$
14. $y = x^2 - 6x - 8$
    $y = 10$

Find the solution set of each system of equations by elimination or by a combination of elimination and substitution. See example 9–5 B.

Example $x^2 + y^2 = 18$
$2x^2 - y^2 = -6$

Solution We can eliminate y by adding members.

$$\begin{aligned} x^2 + y^2 &= 18 \quad (1) \\ 2x^2 - y^2 &= -6 \quad (2) \\ \hline 3x^2 &= 12 \quad \text{Add members} \\ x^2 &= 4 \quad \text{Divide each member by 3} \\ x &= \pm 2 \quad \text{Extract the roots} \end{aligned}$$

Substitute 2 and -2 in equation (1) and solve for y.

Let $x = 2$.	Let $x = -2$.
$x^2 + y^2 = 18$	$x^2 + y^2 = 18$
$(2)^2 + y^2 = 18$	$(-2)^2 + y^2 = 18$
$4 + y^2 = 18$	$4 + y^2 = 18$
$y^2 = 14$	$y^2 = 14$
$y = \pm\sqrt{14}$	$y = \pm\sqrt{14}$

The solutions are $(2, \sqrt{14})$, $(2, -\sqrt{14})$, $(-2, \sqrt{14})$, and $(-2, -\sqrt{14})$ and the solution set is $\{(2, \sqrt{14}), (2, -\sqrt{14}), (-2, \sqrt{14}), (-2, -\sqrt{14})\}$.

15. $x^2 + y^2 = 9$
$x^2 - y^2 = 9$

16. $x^2 - y^2 = 24$
$x^2 + y^2 = 8$

17. $x^2 + 4y^2 = 64$
$x^2 + y^2 = 25$

18. $x^2 + 2y^2 = 22$
$2x^2 + y^2 = 17$

19. $x^2 - 2y^2 = -9$
$x^2 + y^2 = 18$

20. $3x^2 + 4y^2 = 16$
$x^2 - y^2 = 8$

$\boxed{21.}$ $2x^2 - 9y^2 = 18$
$4x^2 + 9y^2 = 36$

22. $3x^2 + 4y^2 = 39$
$5x^2 - 2y^2 = -13$

23. $x^2 + y^2 = 4$
$2x^2 + 3y^2 = 5$

$\boxed{24.}$ $x^2 + y^2 = 10$
$x^2 - 3xy + y^2 = 1$

25. $x^2 + y^2 = 6$
$2x^2 + 3xy + 2y^2 = 21$

26. $x^2 + xy - y^2 = 1$
$x^2 - y^2 = 3$

For each of the following exercises, translate the verbal statements into a system of equations and solve.

Example Find the dimensions of a rectangular plot of ground if the perimeter is 22 meters and the area is 24 square meters.

Solution We use the formulas perimeter $P = 2\ell + 2w$ and area $A = \ell w$, where ℓ is the length and w is the width of the rectangle. Replacing P with 22 and A with 24, we have the system of equations

$$2\ell + 2w = 22$$
$$\ell w = 24$$

Solving the equation $\ell w = 24$ for ℓ, we obtain

$$\ell = \frac{24}{w}$$

Then, the equation $2\ell + 2w = 22$ becomes

$2\left(\frac{24}{w}\right) + 2w = 22$	Replace ℓ with $\frac{24}{w}$
$\frac{48}{w} + 2w = 22$	Multiply in left member
$48 + 2w^2 = 22w$	Multiply each term by w
$2w^2 - 22w + 48 = 0$	Write equation in standard form
$2(w^2 - 11w + 24) = 0$	Factor left member
$2(w - 3)(w - 8) = 0$	
$w - 3 = 0$ or $w - 8 = 0$	Set each factor equal to 0
$w = 3$ or $w = 8$	Solve for w

Substituting for w in the equation $\ell = \frac{24}{w}$, when

$w = 3$	or $w = 8$
$\ell = \frac{24}{3} = 8$	$\ell = \frac{24}{8} = 3$

In either case, the dimensions of the rectangle are 8 meters by 3 meters.

27. Find the dimensions of a rectangle whose area is 120 square inches and whose perimeter is 46 inches.

28. The area of a rectangle is 80 square centimeters and the perimeter is 42 centimeters. What are the dimensions of the rectangle?

$\boxed{29.}$ The sum of two numbers is 16 and the difference between their squares is 32. Find the numbers.

30. The sum of two numbers is 12 and the sum of their squares is 80. Find the numbers.

31. A manufacturer determines that the relationship between the demand (x) and the price (p) for their commodity is $xp = 6$ and the relationship between the supply (x) and price (p) of the same commodity is $4x = 23 - 5p$. The **equilibrium point** is where the supply equals the demand. Solve the system of equations to find the equilibrium point.

32. A piece of cardboard is in the form of a rectangle and has an area of 260 square inches. A square 2 inches on each side is cut out of each corner and an open box is formed by folding up the ends and sides. If the volume of the resulting box is 288 cubic inches, find the length and width of the cardboard. (*Hint:* $V = \ell wh$)

Review exercises

Evaluate the following expressions. See section 1–5.

1. $2x - 3y$ when $x = 3$ and $y = 4$

2. $4x^2 + 2y^2$ when $x = -1$ and $y = 2$

3. Find the solution set of the inequality $3y - 2 \geq 2$. See section 2–5.

Given $P(x) = 2x - 5$ and $Q(x) = x^2 + x - 3$, find

4. $P(-6)$

5. $Q(2)$

6. $P(2) - Q(-1)$

See section 1–5.

7. Find the solution set of the absolute inequality $|2x - 5| \leq 3$. See section 2–5.

Chapter 9 lead-in problem

If a gun is fired upward with an initial velocity of 288 ft/sec, the bullet's height h after t seconds is given by the equation $h = 288t - 16t^2$. Find the maximum height attained by the bullet.

Solution

The path the bullet will follow is a parabola. We want the coordinates (t,h) of the vertex. The first coordinate of the vertex, t, is given by

$$t = -\frac{b}{2a}$$

where $b = 288$ and $a = -16$. Then,

$$\begin{aligned} t &= -\frac{(288)}{2(-16)} && \text{Replace } b \text{ with 288 and } a \text{ with } -16 \\ &= \frac{288}{32} = 9 \end{aligned}$$

Maximum height is attained at 9 seconds. Now we want h.

$$\begin{aligned} h &= 288t - 16t^2 \\ &= 288(9) - 16(9)^2 && \text{Replace } t \text{ with 9} \\ &= 2{,}592 - 1{,}296 \\ &= 1{,}296 \end{aligned}$$

The maximum height attained by the bullet is 1,296 feet.

Chapter 9 summary

1. **Conic sections**—the parabola, the circle, the ellipse, and the hyperbola—are obtained by slicing a cone with a plane in four different ways.
2. The graph of a **quadratic equation** of the form $y = ax^2 + bx + c$ is a parabola opening upward if $a > 0$ and opening downward if $a < 0$.
3. The graph of the quadratic equation of the form $x = ay^2 + by + c$ is a parabola opening right if $a > 0$ and opening left if $a < 0$.
4. The equation of the **circle** with center $C(h,k)$ and having radius r is given by

   $(x - h)^2 + (y - k)^2 = r^2$

   When C is at $(0,0)$, the equation is given by

   $x^2 + y^2 = r^2$
5. The equation of the ellipse with center $C(0,0)$ is given by

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

   where the x-intercepts are a and $-a$ and the y-intercepts are b and $-b$.
6. The equation of the hyperbola with center $C(0,0)$ and having x-intercepts a and $-a$ is given by

   $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

   and having y-intercepts b and $-b$ is given by

   $$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$
7. The equations of the **asymptotes** for the graph of either hyperbola are given by

   $$y = \frac{b}{a}x \text{ and } y = -\frac{b}{a}x$$

   They are the diagonals of a rectangle whose vertices are (a,b), $(a,-b)$, $(-a,b)$, and $(-a,-b)$.

Chapter 9 error analysis

1. Graph of the parabola
   Example: The graph of $y = -3x^2 + 2x - 1$ opens upward since $a = -3$.
   Correct answer: Graph opens downward.
   What error was made? (*see page 404*)
2. Vertex of a parabola
   Example: Given $y = (x + 3)^2 - 2$, the vertex is the point $(3, -2)$.
   Correct answer: $(-3, -2)$
   What error was made? (*see page 403*)
3. Finding the vertex of a parabola
   Example: Given $y = -x^2 + 4x - 3$,
   $$\begin{aligned}\text{Then } y &= -1(x^2 - 4x) - 3\\ &= -1(x^2 - 4x + 4) - 3 - 4\\ &= -1(x - 2)^2 - 7\end{aligned}$$
   The vertex is the point $(2, -7)$.
   Correct answer: $(2,1)$
   What error was made? (*see page 403*)
4. Finding the intercepts of the parabola
   Example: Given $y = 3x^2 - 7x + 2$,
   1. Let $y = 0$
      $0 = 3x^2 - 7x + 2$
      $0 = (3x - 2)(x - 1)$
      $3x - 2 = 0$ or $x - 1 = 0$
      $x = \frac{2}{3}$ or $x = 1$
      x-intercepts, $\left(\frac{2}{3}, 0\right)$ and $(1,0)$
   2. Let $x = 0$,
      $y = 2$
      y-intercept, $(0,2)$

   Correct answer: x-intercepts, $\left(\frac{1}{3}, 0\right)$, $(2,0)$
   y-intercept, $(0,2)$
   What error was made? (*see page 403–4*)
5. The vertex of a parabola
   Example: The vertex of the parabola whose equation is $x = (y + 3)^2 - 5$ is $(-3, -5)$.
   Correct answer: $(-5, -3)$
   What error was made? (*see page 403*)
6. Center and radius of a circle
   Example: Find the center and radius of the circle whose equation is
   $$\begin{aligned}x^2 + y^2 - 6x + 8y - 2 &= 0\\ (x^2 - 6x) + (y^2 + 8y) &= 2\\ (x^2 - 6x + 9) + (y^2 + 8y + 16) &= 2\\ (x - 3)^2 + (y + 4)^2 &= 2\end{aligned}$$
   Center at $(3,4)$ and radius $r = \sqrt{2}$
   Correct answers: Center at $(3, -4)$ and radius $r = 3\sqrt{3}$
   What errors were made? (*see page 416*)
7. Intercepts of an ellipse
   Example: Find the intercepts of the ellipse whose equation is
   $4x^2 + 8y^2 = 4$
   $x^2 + 2y^2 = 1$
   The x-intercepts are $(1,0)$ and $(-1,0)$ and the y-intercepts are $(0,2)$ and $(0,-2)$.
   Correct answers: x-intercepts, $(1,0)$ and $(-1,0)$
   y-intercepts, $\left(0, -\frac{\sqrt{2}}{2}\right)$ and $\left(0, \frac{\sqrt{2}}{2}\right)$
   What error was made? (*see page 421*)
8. Asymptotes of a hyperbola
   Example: The equations of the asymptotes of the hyperbola whose equation is $\frac{x^2}{4} - \frac{y^2}{9} = 1$ are
   $y = -\frac{4}{9}x$ and $y = \frac{4}{9}x$.
   Correct answers: $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$
   What errors were made? (*see page 423*)
9. Identifying conic sections
   Example: The equation $3x + 3y^2 = 4$ is the equation of a circle.
   Correct answer: parabola
   What error was made? (*see page 426*)
10. Factoring the sum of two cubes
    Example: $8a^3 + 27b^3 = (2a)^3 + (3b)^3$
    $= (2a + 3b)(4a^2 - 12ab + 9b^2)$
    Correct answer: $(2a + 3b)(4a^2 - 6ab + 9b^2)$
    What error was made? (*see page 145*)

Chapter 9 critical thinking

Write an algebraic expression for the following relationship. Will such relationships always be true?

$$\left(\frac{1}{3}\right)^2 + \left(1 - \frac{1}{3}\right) = \left(\frac{1}{3}\right) + \left(1 - \frac{1}{3}\right)^2$$

$$(0.7)^2 + (1 - 0.7) = (0.7) + (1 - 0.7)^2$$

Chapter 9 review

[9–1]

Determine the vertex, x-intercept(s), and y-intercept of the following parabolas.

1. $y = -3(x + 1)^2 + 0$
2. $y = -x^2 + 4x + 45$

Graph each of the following equations.

3. $y = x^2 - 2$
4. $y = -x^2 - 3x + 4$
5. $y = x^2 + 3x + 1$

[9–2]

6. $x = y^2 - 1$
7. $x = y^2 - 4y - 5$
8. $x = -y^2 + y + 2$

[9–3]

Write the equation for each of the following circles in the (1) form $(x - h)^2 + (y - k)^2 = r^2$ and (2) form $x^2 + y^2 + Bx + Cy + D = 0$, where B, C, and D are integers.

9. Center $C(-2,5)$ and radius 5
10. Center $C(4,0)$ and radius $\sqrt{11}$
11. Center at the origin and radius 9

Determine the coordinates of the center and the length of the radius of the following circles.

12. $(x - 5)^2 = 36 - (y + 1)^2$
13. $y^2 = 7 - x^2$
14. $x^2 + y^2 + 6x - 8y + 1 = 0$
15. $x^2 + y^2 - 2x + 4y - 3 = 0$
16. $4x^2 + 4y^2 - 16y + 8x - 20 = 0$
17. $x^2 + y^2 - 4y - 15 = 0$

Graph each of the following circles.

18. $x^2 + y^2 = 2$
19. $5x^2 = 20 - 5y^2$
20. $(x - 3)^2 + (y + 4)^2 = 1$
21. $x^2 + y^2 + 6x = 0$
22. $x^2 - 8y = 6x - y^2$
23. $x^2 + y^2 + 6x - 4y - 3 = 0$

[9–4]

Determine the x- and y-intercepts for each given ellipse. Graph the equation.

24. $\dfrac{x^2}{16} + \dfrac{y^2}{100} = 1$
25. $x^2 + \dfrac{y^2}{25} = 1$
26. $16x^2 + 9y^2 = 144$
27. $4x^2 + 8y^2 = 16$
28. $6x^2 + y^2 = 24$
29. $x^2 + 8y^2 = 8$

Determine the x-intercepts, or y-intercepts, and the equations of the asymptotes for the given hyperbola. Graph the equation.

30. $\dfrac{x^2}{36} - \dfrac{y^2}{49} = 1$
31. $\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1$
32. $x^2 - \dfrac{y^2}{36} = 1$
33. $y^2 = 1 + \dfrac{x^2}{9}$
34. $16x^2 - 25y^2 = 400$
35. $x^2 - 9y^2 = -9$

Identify each equation as a parabola, a circle, an ellipse, or a hyperbola. Graph each equation.

36. $x^2 - 16 = y^2$
37. $y + 4x^2 = 8x - 3$
38. $4x^2 = 100 - 25y^2$
39. $9y^2 = 225 - 9x^2$
40. $y^2 = 8 - 2x^2$
41. $y^2 = x - 2y + 3$

[9–5]

Solve the following systems of nonlinear equations.

42. $x^2 + y^2 = 2$; $x + y = 2$
43. $x^2 + y^2 = 25$; $x = 7 - y$
44. $x^2 + y^2 = 5$; $x = 3$
45. $2x^2 - 3y^2 = 4$; $y = -2$
46. $x^2 - y^2 = -1$; $x + y = 3$

Solve each system of equations by elimination or a combination of elimination and substitution.

47. $2x^2 + y^2 = 4$
$x^2 - y^2 = 8$

48. $x^2 - y^2 = 16$
$x^2 + 2y^2 = 25$

49. Find the length and the width of a rectangle whose perimeter is 52 inches and whose area is 153 square inches.

Chapter 9 cumulative test

Given $x = 3$, $y = -5$, and $z = -2$, evaluate the following expressions.

[1–5] 1. $2x - 3y + 4z$

[1–5] 2. $4x^2 + 2y - 5$

[1–5] 3. $4x^2 + y^2 - z^2$

[1–5] 4. Given $C = \frac{5}{9}(F - 32)$, find C when $F = 212$.

[1–5] 5. What is the degree of the polynomial $4x^4 - 3x^2 + 7$?

[3–2] 6. From $4x^2 + 6x - 9$ subtract $-2x^2 - x + 5$.

Given $P(x) = x^2 + 3x - 1$, $Q(x) = 4x - 1$, and $R(x) = x^2 + 3$ find

[1–5] 7. $P(x) - Q(x) + R(x)$

[1–5] 8. $P(x) + Q(x) - R(x)$

[1–5] 9. $P(3)$

[1–5] 10. $Q(-4)$

[1–5] 11. $R\left(\frac{1}{4}\right)$

[3–1] 12. Simplify the expression $x^{2+n}x^{3n-1}$ by performing the indicated multiplication.

Perform the indicated operations.

[3–2] 13. $(2a - 1)(3a^2 + 4a - 1)$

[4–5] 14. $\dfrac{28a^2b^3c^5 - 35ab^3c^3}{7abc}$

[4–5] 15. $(3x^3 - 4x^2 - 5x - 4) \div (x + 2)$

[4–3] 16. $(2x + 9) - \dfrac{x + 3}{x - 5}$

[4–2] 17. $\dfrac{b^2 - 16}{2b + 1} \div (b + 4)$

[4–2] 18. $\dfrac{x^2 - x - 6}{x^2 + x - 12} \cdot \dfrac{x^2 + 3x - 4}{x^2 + 2x - 3}$

[4–4] 19. Simplify the complex rational expression
$$\frac{\frac{1}{y} - \frac{4}{x}}{\frac{2x - 8y}{xy}}.$$

Find the solution set of the following equations.

[4–7] 20. $\dfrac{y}{y - 3} + \dfrac{4}{5} = \dfrac{3}{y - 3}$

[6–3] 21. $3x^2 - 2x + 2 = 0$

[2–4] 22. $|4x - 3| = 7$

[6–6] 23. $x^4 - 3x^2 - 10 = 0$

[6–5] 24. $\sqrt{2x + 1} - 3 = 0$

[7–4] 25. Graph the inequality $3y - 2x \leq 12$.

Find the slope of the given line.

[7–2] 26. Passing through $(-5,6)$ and $(1,2)$

[7–2] 27. Whose equation is $3x - 5y = 10$

[7–3] 28. Determine if the lines $2x + 4y = 3$ and $4x - 8y = 6$ are parallel, perpendicular, or neither.

[7–3] 29. Find the equation of the horizontal line passing through the point $(-1,7)$.

Find the solution set of the following systems of equations.

[8–1] 30. $3x - 4y = 2$
$2x + y = -1$

[8–3] 31. $x + y - z = 1$
$x + y + z = 3$
$-x - y + z = -6$

[8–4] 32. Evaluate $\begin{vmatrix} -3 & 0 & 1 \\ 2 & 4 & 1 \\ -1 & -1 & 0 \end{vmatrix}$.

[9–3] 33. Given the circle $x^2 + y^2 - 6x + 4y - 3 = 0$, find the coordinates of the center and the length of the radius.

[9–1] 34. Find the vertex and intercepts of the parabola $y = 2x^2 - 4x + 3$.

[9–4] 35. Find the x- and y-intercepts of the ellipse $3x^2 = 12 - 2y^2$.

[9–4] 36. Find the intercepts and equations of the asymptotes of the hyperbola
$$\frac{x^2}{9} - \frac{y^2}{4} = 1.$$

Identify each equation as a circle, a parabola, an ellipse, or a hyperbola. Sketch the graph.

[9–4] 37. $4x^2 - y = x + 3$

[9–4] 38. $4y^2 = 2x^2 + 8$

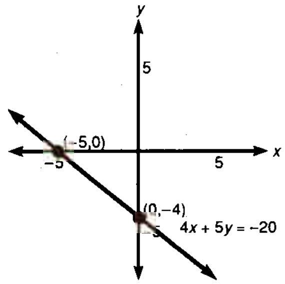
[9–4] 39. $3x^2 + 3y^2 = 12$.

Find the solution set of the following systems of nonlinear equations.

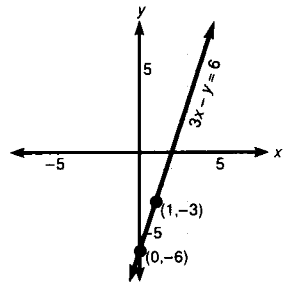
[9–5] 40. $y = x^2$
$y = -2x + 8$

[9–5] 41. $x^2 + y^2 = 7$
$x^2 - y^2 = 1$

30.

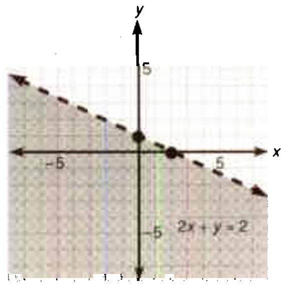


31.



32. $5x + 7y = 28$

33. $x + 4y = 13$ **34.**



35. -31 **36.** $\{(2,0)\}$ **37.** $\left\{\left(\frac{9}{5},-\frac{11}{5}\right)\right\}$

38. $\left\{\left(\frac{3}{17},-\frac{4}{17}\right)\right\}$ **39.** $\left\{\left(-\frac{5}{11},-\frac{7}{11}\right)\right\}$

40. $\left\{\left(-\frac{1}{6},-\frac{2}{3},\frac{1}{6}\right)\right\}$

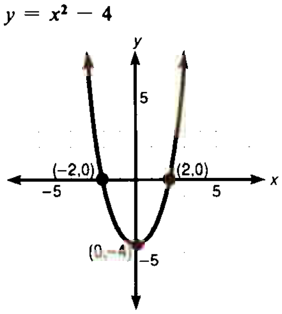
Chapter 9

Exercise 9–1

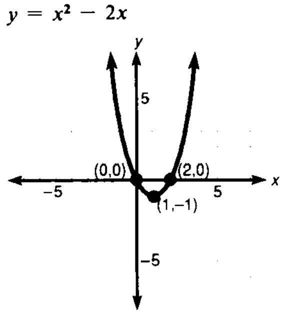
Answers to odd-numbered problems

1. $(3,4)$ **3.** $(0,-16)$ **5.** $(5,0)$ **7.** $(-2,-9)$ **9.** $(1,4)$
11. $\left(\frac{7}{4},-\frac{25}{8}\right)$ **13.** x-intercepts, none; y-intercept, $(0,13)$
15. x-intercepts, $(-4,0)$, $(4,0)$; y-intercept, $(0,-16)$
17. x-intercept, $(5,0)$; y-intercept, $(0,25)$ **19.** x-intercepts, $(-5,0)$, $(1,0)$; y-intercept, $(0,-5)$ **21.** x-intercepts, $(3,0)$, $(-1,0)$; y-intercept, $(0,3)$ **23.** x-intercepts, $(3,0)$, $\left(\frac{1}{2},0\right)$; y-intercept, $(0,3)$

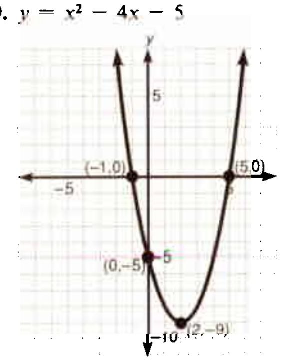
25. $y = x^2 - 4$



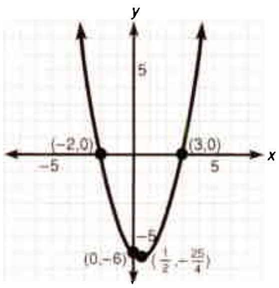
27. $y = x^2 - 2x$



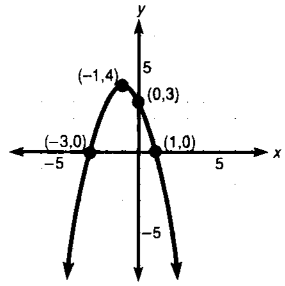
29. $y = x^2 - 4x - 5$



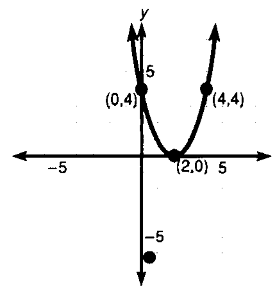
31. $y = x^2 - x - 6$



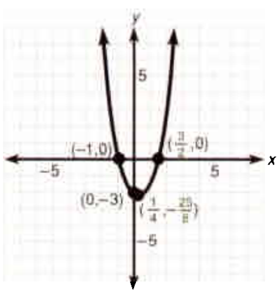
33. $y = -x^2 - 2x + 3$



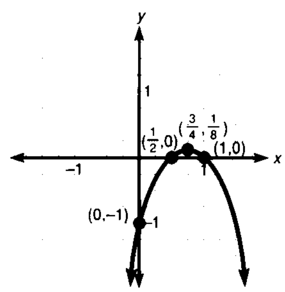
35. $y = x^2 - 4x + 4$



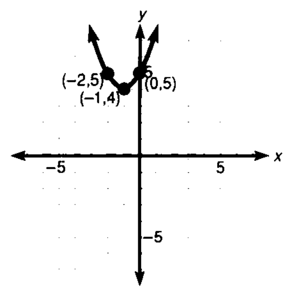
37. $y = 2x^2 - x - 3$



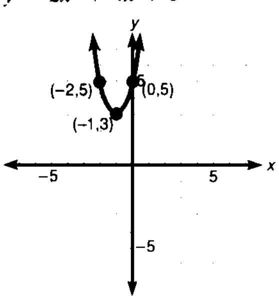
39. $y = -2x^2 + 3x - 1$



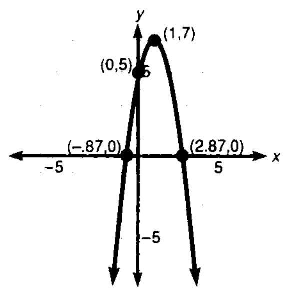
41. $y = x^2 + 2x + 5$



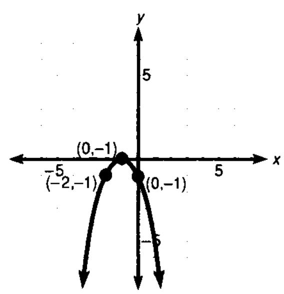
43. $y = 2x^2 + 4x + 5$



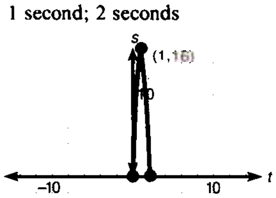
45. $y = -2x^2 + 4x + 5$



47. $y = -x^2 - 2x - 1$



49. 1 second; 2 seconds



51. The maximum height is 144 feet; the arrow will reach the ground in 6 seconds. **53.** 8 dresses must be sold daily. **55.** 5 glasses of Kool-Aid must be sold. **57.** Maximum power $P = 245$ when current $I = 35$. **59.** 28 and 28 **61.** $\ell = w = \frac{21}{2}$ m or $10\frac{1}{2}$ m

63. -16

Solutions to trial exercise problems

3. $y = x^2 - 16$. $y = (x - 0)^2 - 16$. The vertex is at $(0,-16)$ and since $a = 1$, $a > 0$, the parabola opens up and the vertex is the lowest point.

9. $y = -x^2 + 2x + 3 = -1(x^2 - 2x + 1) + 3 + 1 = -1(x - 1)^2 + 4$, vertex at $(1,4)$

11. $y = 2x^2 - 7x + 3 = 2\left(x^2 - \frac{7}{2}x\right) + 3$

$= 2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) + 3 - \frac{98}{16}$

$= 2\left(x - \frac{7}{4}\right)^2 - \frac{25}{8}$

Vertex at $\left(\frac{7}{4}, -\frac{25}{8}\right)$

13. $y = (x - 3)^2 + 4$

(1) Let $x = 0$, then $y = (0 - 3)^2 + 4 = (-3)^2 + 4 = 13$; y-intercept, $(0,13)$

(2) Let $y = 0$, then $0 = (x - 3)^2 + 4 = x^2 - 6x + 9 + 4$
$= x^2 - 6x + 13$

Using the quadratic formula,

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$

$= \frac{6 \pm \sqrt{36 - 52}}{2}$

$= \frac{6 \pm \sqrt{-16}}{2}$

There are no x-intercepts since $\sqrt{-16}$ is not a real number.

18. $y = (x + 6)^2$

(1) Let $x = 0$, then $y = (0 + 6)^2 = 6^2 = 36$; y-intercept, $(0,36)$

(2) Let $y = 0$, then $0 = (x + 6)^2$ and $x = -6$; x-intercept, $(-6,0)$

21. $y = -x^2 + 2x + 3$

(1) Let $x = 0$, then $y = 3$; y-intercept, $(0,3)$

(2) Let $y = 0$, then $0 = -x^2 + 2x + 3$; $0 = x^2 - 2x - 3$; $0 = (x - 3)(x + 1)$; $x = 3$ or $x = -1$; x-intercepts, $(3,0)$, $(-1,0)$.

27. $y = x^2 - 2x$

(1) $a > 0$, parabola opens up

(2) $c = 0$, so y-intercept is $(0,0)$

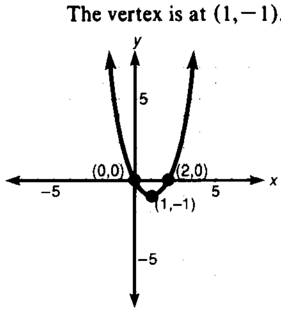
(3) Let $y = 0$, $0 = x^2 - 2x$
$= x(x - 2)$

$x = 0$ or $x - 2 = 0$
$x = 2$

The x-intercepts are $(0,0)$ and $(2,0)$.

(4) $y = x^2 - 2x$
$= (x^2 - 2x + 1) - 1$
$= (x - 1)^2 + (-1)$

The vertex is at $(1,-1)$.



34. $y = -x^2 + 7x - 6$

(1) Since $a < 0$, the parabola opens down.

(2) Since $c = -6$, the y-intercept is $(0,-6)$.

(3) Let $y = 0$, $0 = -x^2 + 7x - 6$
$= x^2 - 7x + 6$
$= (x - 6)(x - 1)$

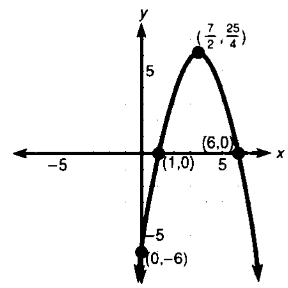
$x - 6 = 0$ or $x - 1 = 0$
$x = 6$ or $x = 1$

The x-intercepts are $(6,0)$, $(1,0)$.

(4) $y = -(x^2 - 7x \quad) - 6$

$= -\left(x^2 - 7x + \frac{49}{4}\right) - 6 + \frac{49}{4} = -\left(x - \frac{7}{2}\right)^2 + \frac{25}{4}$

The vertex is at $\left(\frac{7}{2}, \frac{25}{4}\right)$.



52. $P = -x^2 + 100x - 1{,}000$
$= -1(x^2 - 100x) - 1{,}000$
$= -1(x^2 - 100x + 2{,}500) - 1{,}000 + 2{,}500$
$= -1(x - 50)^2 + 1{,}500$

Since the vertex is at $(50,1{,}500)$, then 50 units must be produced to attain maximum profit.

59. Let x = one number and $56 - x$ = the other number.

Then $y = x(56 - x) = 56x - x^2$
$= -x^2 + 56x$
$= -1(x^2 - 56x + 784) + 784$
$= -1(x - 28)^2 + 784$

The vertex is at $(28,784)$ and the numbers are 28 and $56 - 28 = 28$.

Review exercises

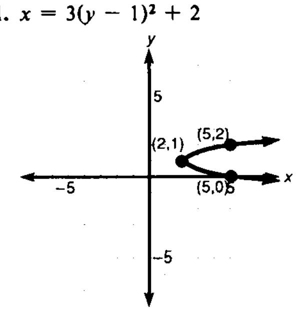
1. $(3y - 1)^2$ **2.** $(2x - 1)(2x + 3)$ **3.** $6(y + 2x)(y - 2x)$

4. $\frac{-y^2 + 9y}{(y + 5)(y - 5)}$ **5.** $12xy$ **6.** $\frac{x - 3}{x + 4}$

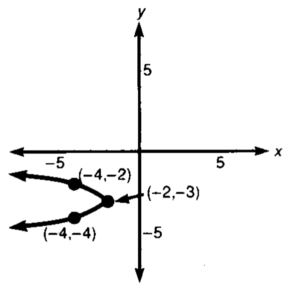
Exercise 9–2

Answers to odd-numbered problems

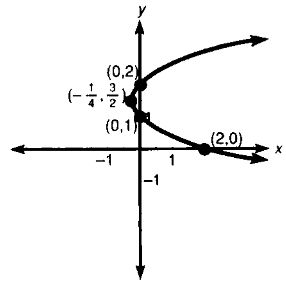
1. $x = 3(y - 1)^2 + 2$



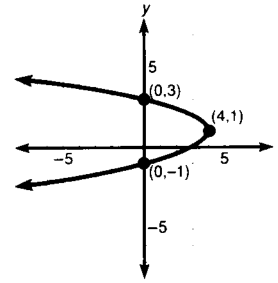
3. $x = -2(y + 3)^2 - 2$



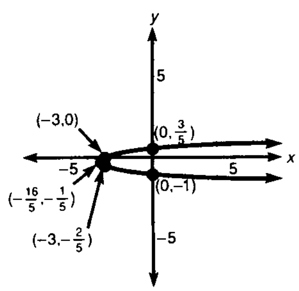
5. $x = y^2 - 3y + 2$



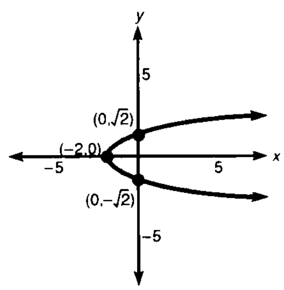
7. $x = -y^2 + 2y + 3$



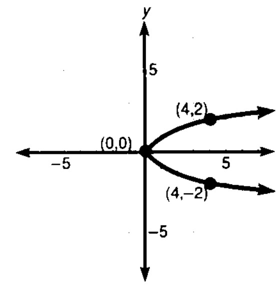
9. $x = 5y^2 + 2y - 3$



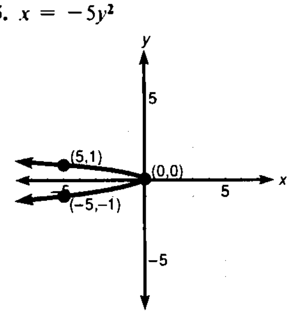
11. $x = y^2 - 2$



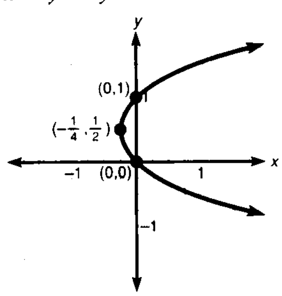
13. $x = y^2$



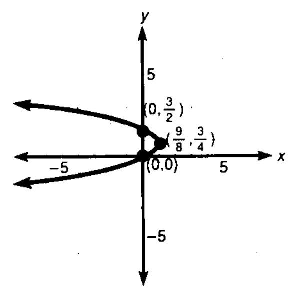
15. $x = -5y^2$



17. $x = y^2 - y$

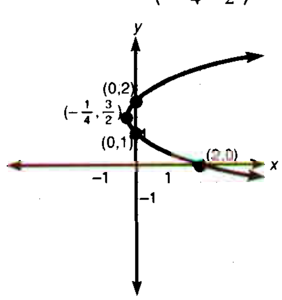


19. $x = -2y^2 + 3y$

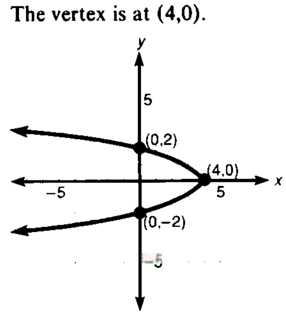


Solutions to trial exercise problems

5. $x = y^2 - 3y + 2$
   (1) Since $a > 0$, parabola opens right.
   (2) Since $c = 2$, the x-intercept is (2,0).
   (3) Let $x = 0$, then $0 = y^2 - 3y + 2$
   $0 = (y - 2)(y - 1)$
   $y - 2 = 0$ or $y - 1 = 0$
   $y = 2$ or $y = 1$
   The y-intercepts are (0,2), (0,1).
   (4) $x = (y^2 - 3y \quad) + 2$
   $= \left(y^2 - 3y + \frac{9}{4}\right) + 2 - \frac{9}{4} = \left(y - \frac{3}{2}\right)^2 - \frac{1}{4}$
   The vertex is at $\left(-\frac{1}{4}, \frac{3}{2}\right)$



12. $x = -y^2 + 4$
   (1) Since $a < 0$, parabola opens left.
   (2) Since $c = 4$, the x-intercept is (4,0).
   (3) Let $x = 0$, $0 = -y^2 + 4$
   $0 = y^2 - 4$
   $0 = (y + 2)(y - 2)$
   $y + 2 = 0$ or $y - 2 = 0$
   $y = -2$ or $y = 2$
   The y-intercepts are (0,−2), (0,2).
   (4) $x = -(y - 0)^2 + 4$
   The vertex is at (4,0).



Review exercises

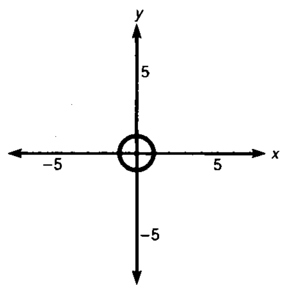
1. $\{(1,2)\}$ **2.** $\frac{a^6}{8b^9}$ **3.** $\frac{1}{8x^{24}}$ **4.** x-intercept, (4,0); y-intercept, (0,2) **5.** x-intercept, (3,0); y-intercept, (0,3) **6.** $\sqrt{41}$
7. $\sqrt{(x-6)^2+(y+5)^2}$ **8.** $\sqrt{(x-h)^2+(y-k)^2}$

Exercise 9–3

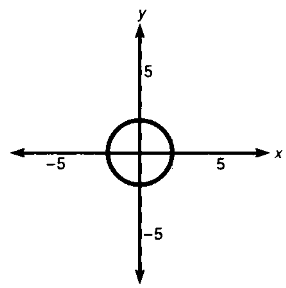
Answers to odd-numbered problems

1. $(x-1)^2+(y-2)^2=4$ **3.** $(x-4)^2+(y+3)^2=6$
5. $x^2+y^2=9$ **7.** $x^2+y^2+10x-4y+28=0$
9. $x^2+y^2=36$ **11.** $C(3,2), r=7$ **13.** $C(5,-3), r=2\sqrt{2}$
15. $C(-1,-9), r=5$ **17.** $C(0,0); r=6$ **19.** $C(0,0), r=\sqrt{2}$
21. $C(-2,3), r=6$ **23.** $C(-3,4), r=5$
25. $C(-2,3), r=5\sqrt{2}$ **27.** $C(-3,1), r=5$

29.



31.



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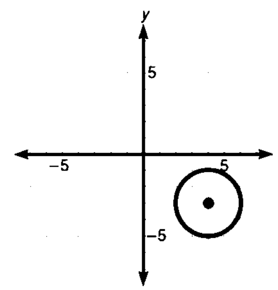

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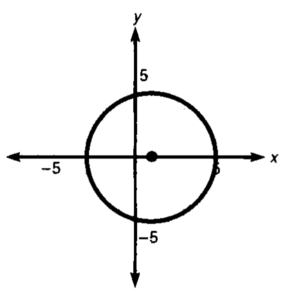
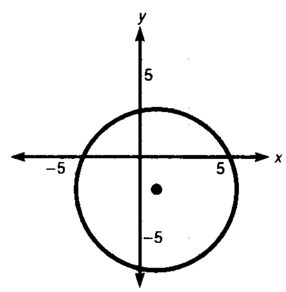
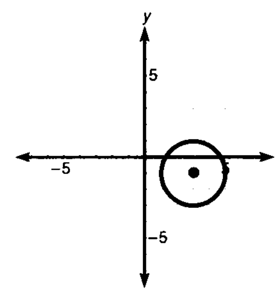


37.



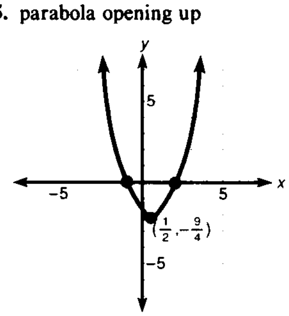
39.



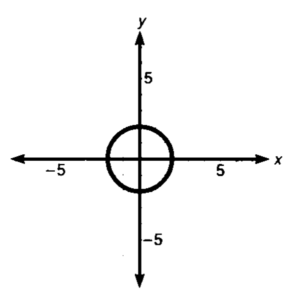
41. $r=\sqrt{(x-h)^2+(y-k)^2}$
Square both members.
$(x-h)^2+(y-k)^2=r^2$

43. $(x+2)^2+y^2=4$

45. parabola opening up



47. circle with center (0,0) and $r=2$



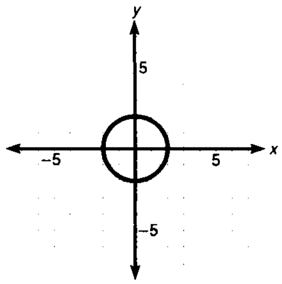
Solutions to trial exercise problems

7. $(x + 5)^2 + (y - 2)^2 = 1^2$; $(x^2 + 10x + 25) + (y^2 - 4y + 4) = 1$; $x^2 + y^2 + 10x - 4y + 28 = 0$

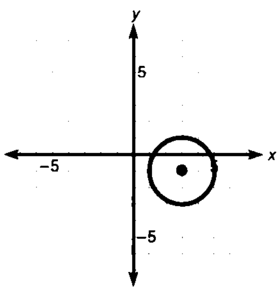
21. $x^2 + y^2 + 4x - 6y - 23 = 0$; $(x^2 + 4x + 4) + (y^2 - 6y + 9) = 23 + 4 + 9$; $(x + 2)^2 + (y - 3)^2 = 36$; $[x - (-2)]^2 + (y - 3)^2 = 6^2$; $C(-2,3)$, $r = 6$

25. $2x^2 + 2y^2 + 8x - 12y = 74$; $x^2 + y^2 + 4x - 6y = 37$; $(x^2 + 4x + 4) + (y^2 - 6y + 9) = 37 + 4 + 9$; $(x + 2)^2 + (y - 3)^2 = 50$; $C(-2,3)$, $r = 5\sqrt{2}$

31. $3x^2 + 3y^2 = 12$; $x^2 + y^2 = 4$; $C(0,0)$, $r = 2$



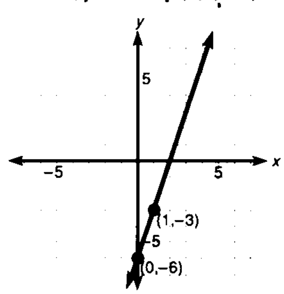
39. $2x^2 + 2y^2 - 12x + 4y = -12$; $x^2 + y^2 - 6x + 2y = -6$; $(x^2 - 6x + 9) + (y^2 + 2y + 1) = -6 + 9 + 1$; $(x - 3)^2 + (y + 1)^2 = 4$; $C(3,-1)$, $r = 2$



Review exercises

1. -58

2. $m = 3$; y-intercept, $(0,-6)$



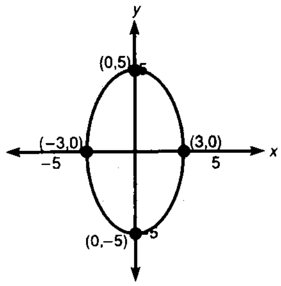
3. $-6y\sqrt{2xy}$

4. $P(-1) = 14$ 5. $3x + 2y = 4$

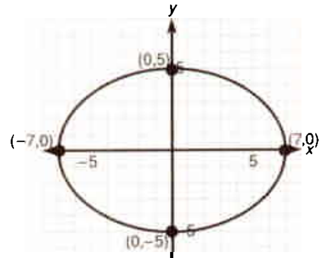
Exercise 9–4

Answers to odd-numbered problems

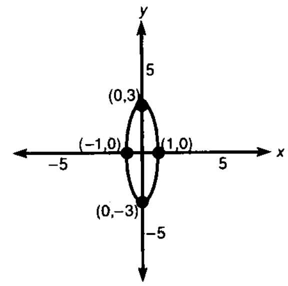
1. x, $(\pm 3,0)$; y, $(0,\pm 5)$



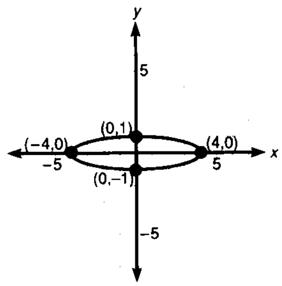
3. x, $(\pm 7,0)$; y, $(0,\pm 5)$



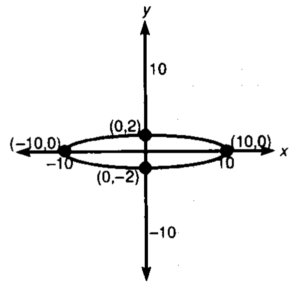
5. x, $(\pm 1,0)$; y, $(0,\pm 3)$



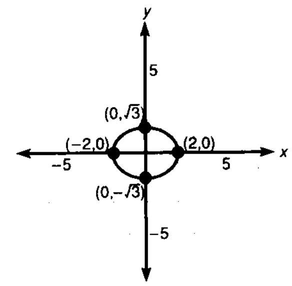
7. x, $(\pm 4,0)$; y, $(0,\pm 1)$



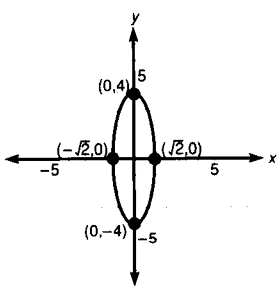
9. x, $(\pm 10,0)$; y, $(0,\pm 2)$



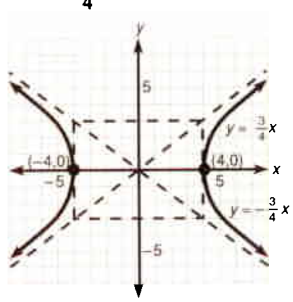
11. x, $(\pm 2,0)$; y, $(0,\pm \sqrt{3})$



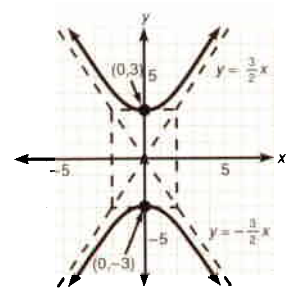
13. x, $(\pm \sqrt{2},0)$; y, $(0,\pm 4)$



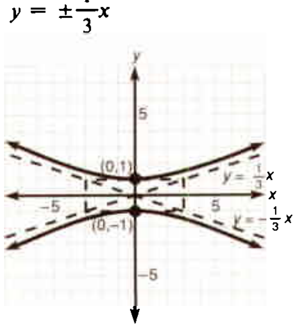
15. x, $(\pm 4,0)$; asymptotes,
$y = \pm\frac{3}{4}x$



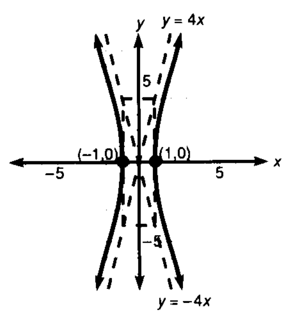
17. y, $(0,\pm 3)$; asymptotes,
$y = \pm\frac{3}{2}x$



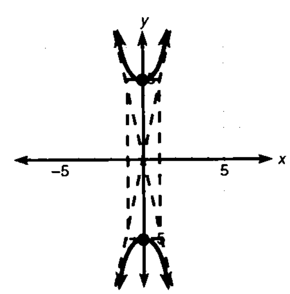
19. y, $(0,\pm 1)$; asymptotes,
$y = \pm\frac{1}{3}x$



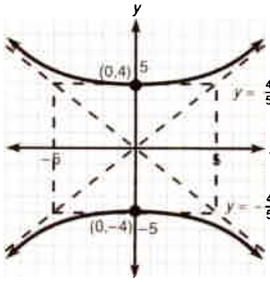
21. x, $(\pm 1,0)$; asymptotes, $y = \pm 4x$



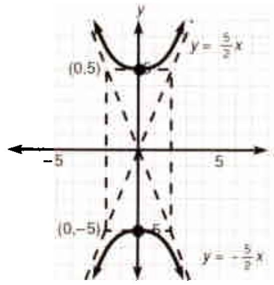
23. y, $(0,\pm 5)$; asymptotes, $y = \pm 5x$



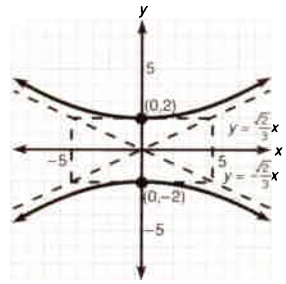
25. y, $(0, \pm 4)$; asymptotes, $y = \pm\frac{4}{5}x$



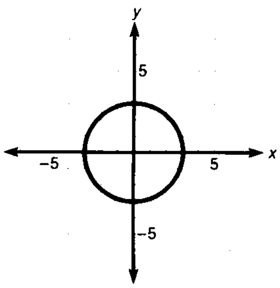
27. y, $(0, \pm 5)$; asymptotes, $y = \pm\frac{5}{2}x$



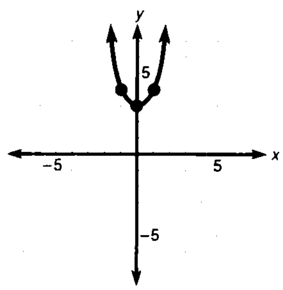
29. y, $(0, \pm 2)$; asymptotes, $y = \pm\frac{\sqrt{2}}{3}x$



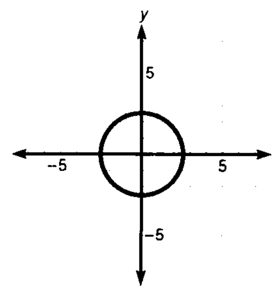
31. $x^2 + y^2 = 9$; circle



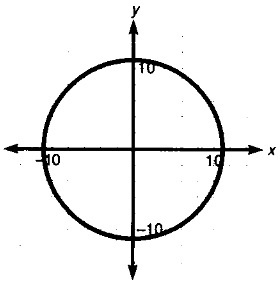
33. $x^2 + 3 = y$; parabola opening up



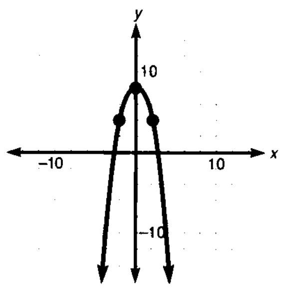
35. $x^2 + y^2 = \frac{25}{4}$; circle



37. $x^2 + y^2 = 121$; circle

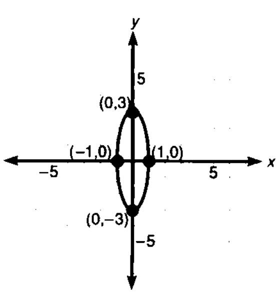


39. $y = -x^2 + 8$; parabola opening down

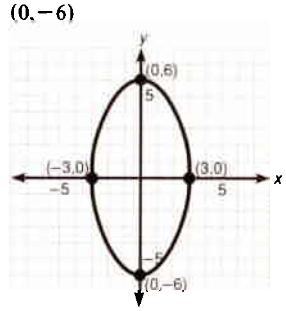


Solutions to trial exercise problems

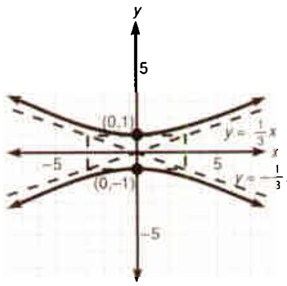
5. $x^2 + \frac{y^2}{9} = 1$; $b^2 = 1$, so $b = 1$; $a^2 = 9$, so $a = 3$;
x-intercepts, $(1,0)$, $(-1,0)$; y-intercepts, $(0,3)$, $(0,-3)$



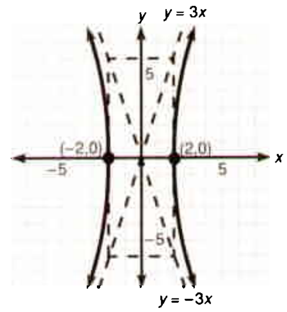
8. $36x^2 + 9y^2 = 324$; $\frac{x^2}{9} + \frac{y^2}{36} = 1$; $b^2 = 9$, $b = 3$;
$a^2 = 36$, $a = 6$; x-intercepts, $(3,0)$, $(-3,0)$; y-intercepts, $(0,6)$, $(0,-6)$



19. $y^2 - \frac{x^2}{9} = 1$; $a^2 = 9$, $a = 3$; $b^2 = 1$, $b = 1$;
y-intercepts, $(0,1)$, $(0,-1)$;
asymptotes, $y = \frac{1}{3}x$ and $y = -\frac{1}{3}x$



22. $9x^2 - y^2 = 36$; $\frac{x^2}{4} - \frac{y^2}{36} = 1$; $a^2 = 4$, $a = 2$; $b^2 = 36$,
$b = 6$; x-intercepts, $(2,0)$, $(-2,0)$;
asymptotes, $y = 3x$ and $y = -3x$



33. Since $x^2 + 3 = y$ has one variable linear and the other squared, it is a parabola.

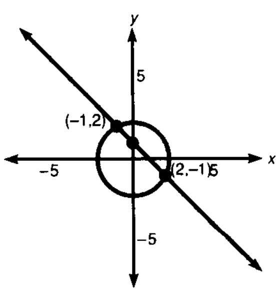
Review exercises

1. $\frac{y+1}{y-3}$ **2.** $\frac{-10z^2 - 16z}{(3z+2)(2z-1)}$ **3.** $\frac{xy}{y+x}$ **4.** $\{-\sqrt{5}, \sqrt{5}\}$
5. $\left\{0, \frac{4}{5}\right\}$ **6.** $-2\sqrt{3} - 3\sqrt{2}$ **7.** $27 + \sqrt{15}$

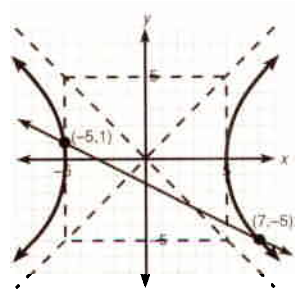
Exercise 9–5

Answers to odd-numbered problems

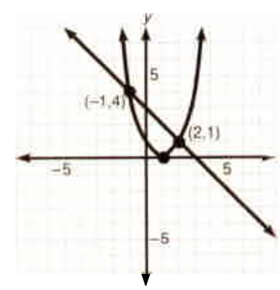
1. $\{(2,-1), (-1,2)\}$



3. $\{(7,-5), (-5,1)\}$



5. $\{(2,1), (-1,4)\}$



7. $\left\{\left(-\frac{2}{9}, -\frac{22}{9}\right), (2,2)\right\}$ **9.** $\{(4,1), (-4,-1), (i,-4i), (-i,4i)\}$
11. $\{(2,2), (-2,-2)\}$ **13.** $\left\{\left(\frac{\sqrt{6}}{2}, -1\right), \left(-\frac{\sqrt{6}}{2}, -1\right)\right\}$
15. $\{(3,0), (-3,0)\}$ **17.** $\{(2\sqrt{3}, \sqrt{13}), (2\sqrt{3}, -\sqrt{13}), (-2\sqrt{3}, \sqrt{13}), (-2\sqrt{3}, -\sqrt{13})\}$
19. $\{(3,3), (3,-3), (-3,3), (-3,-3)\}$ **21.** $\{(3,0), (-3,0)\}$
23. $\{(-\sqrt{7}, -i\sqrt{3}), (-\sqrt{7}, i\sqrt{3}), (\sqrt{7}, -i\sqrt{3}), (\sqrt{7}, i\sqrt{3})\}$
25. $\{(\sqrt{3}, \sqrt{3}), (-\sqrt{3}, -\sqrt{3})\}$ **27.** 15 in. by 8 in. **29.** 9 and 7
31. $x = 2$ when $p = 3$; $\left(x = \frac{15}{4} \text{ when } p = \frac{8}{5}\text{; not logical}\right)$

Solutions to trial exercise problems

9. $x^2 - y^2 = 15$
$xy = 4$

(1) $y = \dfrac{4}{x}$ and substituting,

$$x^2 - \left(\frac{4}{x}\right)^2 = 15$$
$$x^2 - \frac{16}{x^2} = 15$$
$$x^4 - 15x^2 - 16 = 0$$
$$(x^2 - 16)(x^2 + 1) = 0$$
$$(x + 4)(x - 4)(x^2 + 1) = 0$$

Then $x = 4$ or $x = -4$; $x = \sqrt{-1} = i$ or $x = -\sqrt{-1} = -i$

(2) $y = \dfrac{4}{x}$, so $y = \dfrac{4}{4} = 1$ or $y = \dfrac{4}{-4} = -1$;

or $y = \dfrac{4}{i} = -4i$ or $y = \dfrac{4}{-i} = 4i$

The solution set is $\{(4,1), (-4,-1), (i,-4i), (-i, 4i)\}$.

21.
$$\begin{aligned} 2x^2 - 9y^2 &= 18 \\ \underline{4x^2 + 9y^2} &= \underline{36} \\ 6x^2 \qquad &= 54 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

(1) When $x = 3$
$$\begin{aligned} 2(3)^2 - 9y^2 &= 18 \\ 18 - 9y^2 &= 18 \\ -9y^2 &= 0 \\ y &= 0 \end{aligned}$$

(2) When $x = -3$
$$\begin{aligned} 2(-3)^2 - 9y^2 &= 18 \\ 18 - 9y^2 &= 18 \\ -9y^2 &= 0 \\ y &= 0 \end{aligned}$$

The solution set is $\{(3,0), (-3,0)\}$.

24. $x^2 + y^2 = 10$
$\underline{x^2 - 3xy + y^2 = 1}$

(times -1)
$$\begin{aligned} -x^2 \qquad - y^2 &= -10 \\ \underline{x^2 - 3xy + y^2} &= \underline{1} \\ -3xy &= -9 \\ xy &= 3 \\ y &= \frac{3}{x} \end{aligned}$$

(2) Substituting in $x^2 + y^2 = 10$
$$x^2 + \left(\frac{3}{x}\right)^2 = 10$$
$$x^2 + \frac{9}{x^2} = 10$$
$$x^4 + 9 = 10x^2$$
$$x^4 - 10x^2 + 9 = 0$$
$$(x^2 - 9)(x^2 - 1) = 0$$

So $x^2 - 9 = 0$; $x^2 = 9$; $x = \pm 3$
$x^2 - 1 = 0$; $x^2 = 1$; $x = \pm 1$

When $x = 3$, $y = \dfrac{3}{3} = 1$
$x = -3$, $y = \dfrac{3}{-3} = -1$
$x = 1$, $y = \dfrac{3}{1} = 3$
$x = -1$, $y = \dfrac{3}{-1} = -3$

Therefore
the solution set is $\{(3,1), (-3,-1), (1,3), (-1,-3)\}$.

29. Let $x =$ the larger number.
Let $y =$ the smaller number.
Then $x + y = 16$
$x^2 - y^2 = 32$

(1) Now $y = 16 - x$, and substituting, $x^2 - (16 - x)^2 = 32$
$$x^2 - 256 + 32x - x^2 = 32$$
$$32x = 288$$
$$x = 9$$

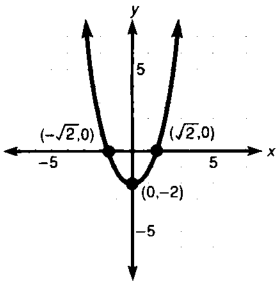
(2) Substituting, $9 + y = 16$, so $y = 7$.
The numbers are 9 and 7.

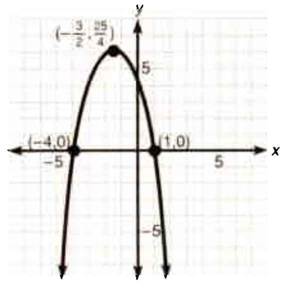
Review exercises

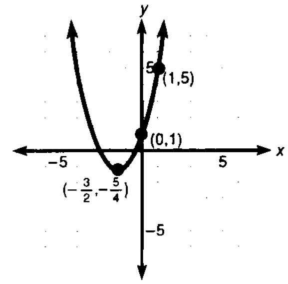
1. -6 **2.** 12 **3.** $\left\{y \mid y \geq \frac{4}{3}\right\} = \left[\frac{4}{3}, \infty\right)$ **4.** -17 **5.** 3 **6.** 2
7. $\{x \mid 1 \leq x \leq 4\} = [1,4]$

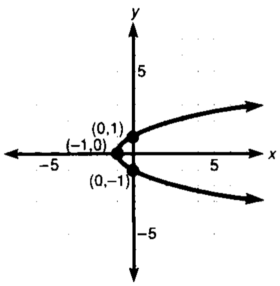
Chapter 9 review

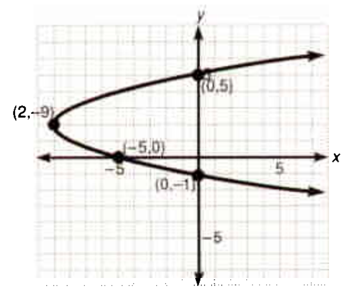
1. vertex, $(-1,0)$; x-intercept, -1; y-intercept, -3
2. vertex, $(2,49)$; x-intercept, 9 and -5; y-intercept, 45

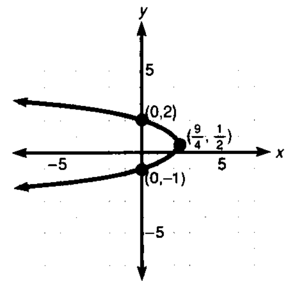
3.


4.


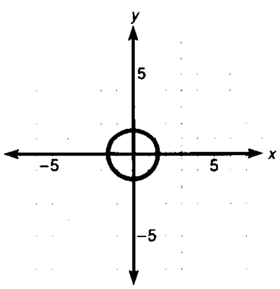
5.


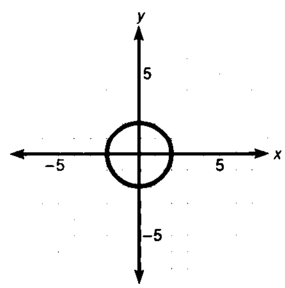
6.


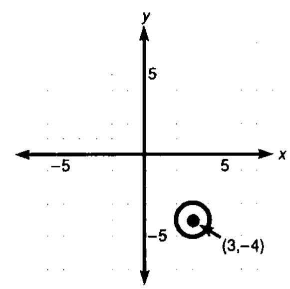
7.


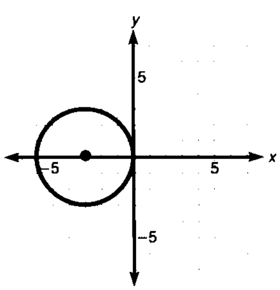
8.


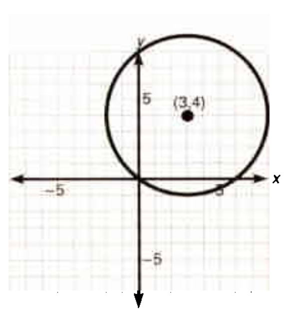
9. a. $(x+2)^2+(y-5)^2=25$ **b.** $x^2+y^2+4x-10y+4=0$
10. a. $(x-4)^2+y^2=11$ **b.** $x^2+y^2-8x+5=0$
11. a. $x^2+y^2=81$ **b.** $x^2+y^2-81=0$
12. $C(5,-1)$, $r=6$ **13.** $C(0,0)$, $r=\sqrt{7}$
14. $C(-3,4)$, $r=2\sqrt{6}$ **15.** $C(1,-2)$; $r=2\sqrt{2}$
16. $C(-1,2)$; $r=\sqrt{10}$ **17.** $C(0,2)$, $r=\sqrt{19}$

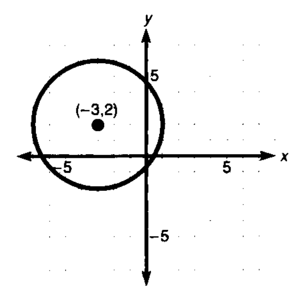
18.


19.


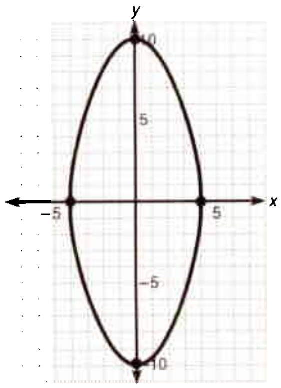
20.


21.


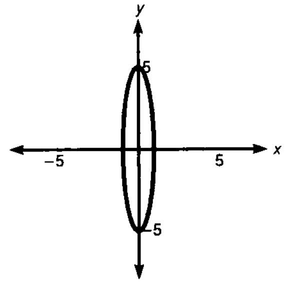
22.


23.


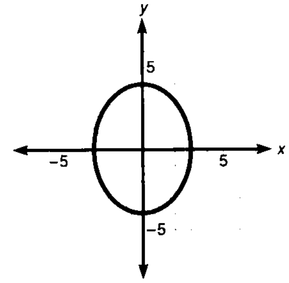
24. x, $(\pm 4,0)$; y, $(0,\pm 10)$



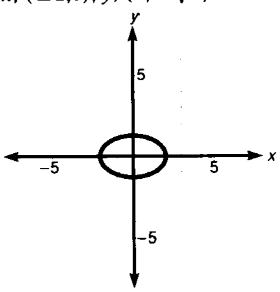
25. x, $(\pm 1,0)$; y, $(0,\pm 5)$



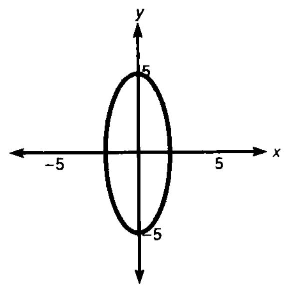
26. x, $(\pm 3,0)$; y, $(0,\pm 4)$



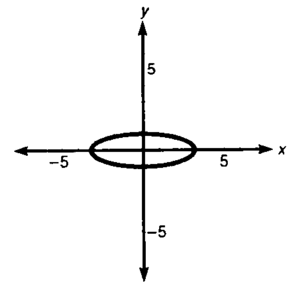
27. x, $(\pm 2,0)$; y, $(0,\pm \sqrt{2})$



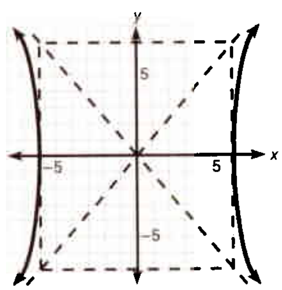
28. x, $(\pm 2,0)$; y, $(0,\pm 2\sqrt{6})$



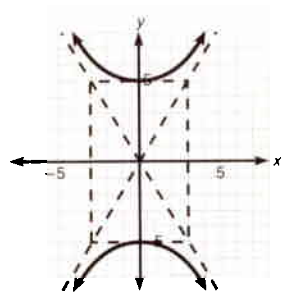
29. x, $(\pm 2\sqrt{2},0)$; y, $(0,\pm 1)$



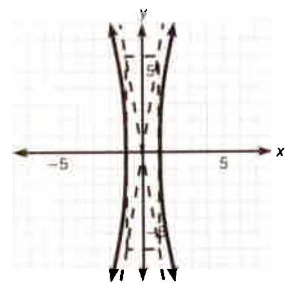
30. x, $(\pm 6,0)$; asymptotes, $y = \pm\dfrac{7}{6}x$



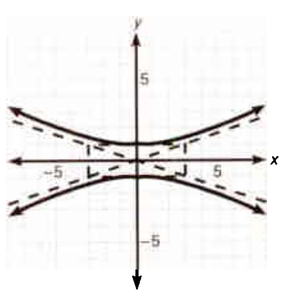
31. y, $(0,\pm 5)$; asymptotes, $y = \pm\dfrac{5}{3}x$



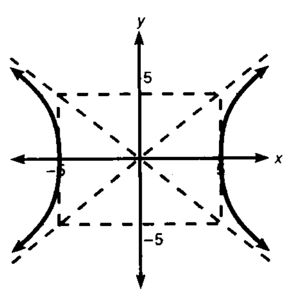
32. x, $(\pm 1,0)$; asymptotes, $y = \pm 6x$



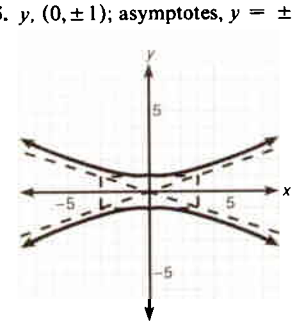
33. y, $(0,\pm 1)$; asymptotes, $y = \pm\dfrac{1}{3}x$



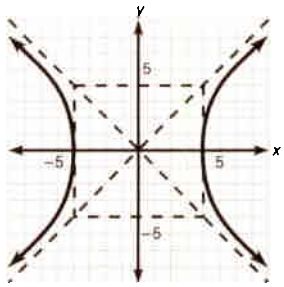
34. x, $(\pm 5,0)$; asymptotes, $y = \pm\dfrac{4}{5}x$



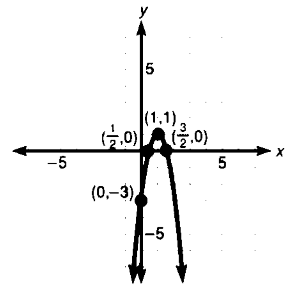
35. y, $(0,\pm 1)$; asymptotes, $y = \pm\dfrac{1}{3}x$



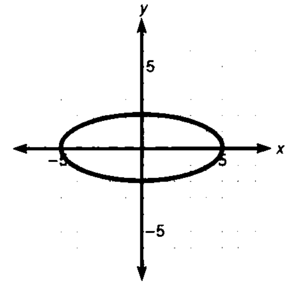
36. hyperbola



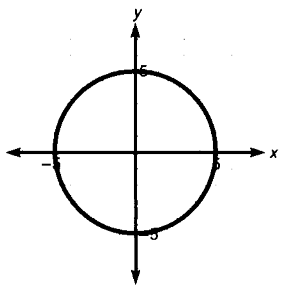
37. parabola opening down



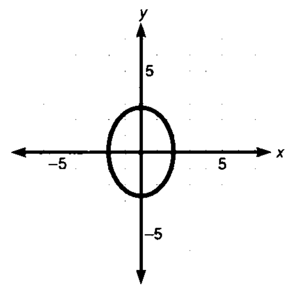
38. ellipse



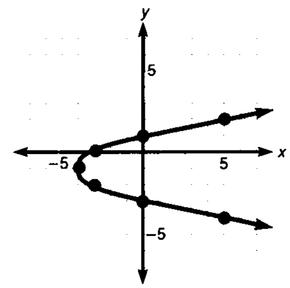
39. circle



40. circle



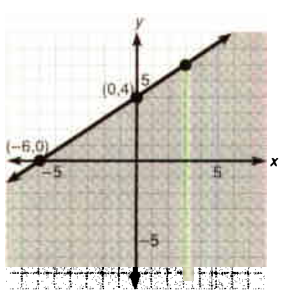
41. parabola opening right



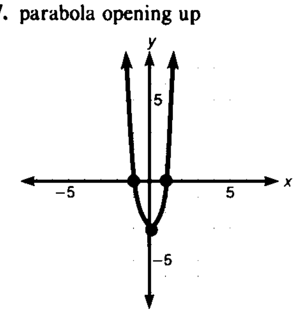
42. $\{(1,1)\}$ **43.** $\{(4,3), (3,4)\}$ **44.** $\{(3,2i), (3,-2i)\}$
45. $\{(2\sqrt{2},-2), (-2\sqrt{2},-2)\}$ **46.** $\left\{\left(\frac{4}{3},\frac{5}{3}\right)\right\}$
47. $\{(2,2i), (2,-2i), (-2,2i), (-2,-2i)\}$
48. $\{(\sqrt{19},\sqrt{3}), (\sqrt{19},-\sqrt{3}), (-\sqrt{19},\sqrt{3}), (-\sqrt{19},-\sqrt{3})\}$
49. 9 in. by 17 in.

Chapter 9 cumulative test

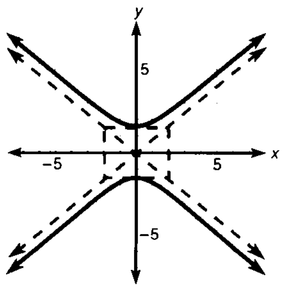
1. 13 **2.** 21 **3.** 57 **4.** $C = 100$ **5.** 4th degree
6. $6x^2 + 7x - 14$ **7.** $2x^2 - x + 3$ **8.** $7x - 5$ **9.** 17
10. -17 **11.** $\frac{49}{16}$ **12.** x^{4n+1} **13.** $6a^3 + 5a^2 - 6a + 1$
14. $4ab^2c^4 - 5b^2c^2$ **15.** $3x^2 - 10x + 15 + \frac{-34}{x+2}$
16. $\frac{2x^2 - 2x - 48}{x - 5}$ **17.** $\frac{4 - b}{2b + 1}$ **18.** $\frac{x + 2}{x + 3}$ **19.** $\frac{1}{2}$
20. $\emptyset$; 3 is extraneous **21.** $\left\{\frac{1 + i\sqrt{5}}{3}, \frac{1 - i\sqrt{5}}{3}\right\}$ **22.** $\left\{\frac{5}{2}, -1\right\}$
23. $\{\sqrt{5}, -\sqrt{5}, i\sqrt{2}, -i\sqrt{2}\}$ **24.** $\{4\}$
25.



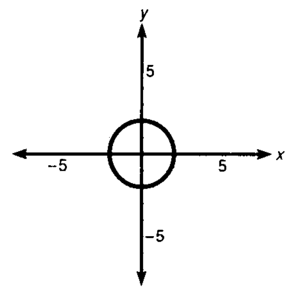
26. $m = -\frac{2}{3}$ **27.** $m = \frac{3}{5}$ **28.** neither **29.** $y = 7$
30. $\left\{\left(-\frac{2}{11}, -\frac{7}{11}\right)\right\}$ **31.** $\emptyset$; inconsistent **32.** -1
33. $C(3,-2)$, $r = 4$
34. vertex, (1,1); y-intercept, (0,3); no x-intercepts
35. x-intercepts, (2,0) and $(-2,0)$; y-intercepts, $(0,\sqrt{6})$ and $(0,-\sqrt{6})$
36. x-intercepts, (3,0) and $(-3,0)$; asymptotes, $y = \pm\frac{2}{3}x$
37. parabola opening up



38. hyperbola, $\frac{y^2}{2} - \frac{x^2}{4} = 1$



39. circle, $x^2 + y^2 = 4$



40. $\{(-4,16), (2,4)\}$
41. $\{(-2,-\sqrt{3}), (-2,\sqrt{3}), (2,-\sqrt{3}), (2,\sqrt{3})\}$

Chapter 10

Exercise 10-1

Answers to odd-numbered problems

1. domain $= \{8,5,9,6\}$, range $= \{0,3,4\}$ **3.** domain $= \{-4,1\}$, range $= \{1,2,3,9\}$ **5.** domain $= \{6,1,2,3\}$, range $= \{-1,1\}$ **7.** domain $= \{5,6,7\}$, range $= \{3,-4\}$ **9.** domain $= \{x|-2 \le x \le 2\}$, range $= \{y|-2 \le y \le 2\}$ **11.** domain $= \{x|x \in R\}$, range $= \{y|y \le 3\}$ **13.** domain $= \{x|x \in R\}$, range $= \{y|y \in R\}$ **15.** domain $= \{x|x \le -2 \text{ or } x \ge 2\}$, range $= \{y|y \in R\}$ **17.** function **19.** not a function because two of the ordered pairs have the same first component (2,3) and (2,−7) **21.** not a function because two of the ordered pairs have the same first component (−1,2) and (−1,6) **23.** function **25.** function **27.** function **29.** not a function because two of the ordered pairs have the same first component (4,−2) and (4,2) **31.** function **33.** not a function since two ordered pairs have the same first component, (−10,0) and (−10,1) **35.** not a function since two ordered pairs have the same first component, (3,1) and (3,4) **37.** domain $= \{-3,-1,0,1,3\}$ **39.** domain $= \{-5,-3,0,3,5\}$ **41.** domain $= \{-5,-1,1,2,4\}$ **43.** domain $= \{x|x \in R\}$ **45.** domain $= \{x|x \in R\}$ **47.** domain $= \{x|x \in R, x \neq 0\}$ **49.** domain $= \left\{x|x \in R, x \neq -\frac{3}{2}\right\}$ **51.** domain $= \left\{x|x \ge -\frac{4}{3}\right\}$ **53.** function since any vertical line intersects the graph at only one point **55.** function since any vertical line intersects the graph at only one point **57.** not a function since at least one vertical line intersects at more than one point on the graph **59.** not a function since at least one vertical line intersects at more than one point on the graph

Solutions to trial exercise problems

5. domain is the set of first components $= \{6,1,2,3\}$, range is the set of second components $= \{-1,1\}$ **9.** domain is set of all values of $x = \{x|-2 \le x \le 2\}$, range is the set of all values of $y = \{y|-2 \le y \le 2\}$ **22.** does define a function since no two ordered pairs have the same first component **31.** defines a function—no two ordered pairs have the same first component **46.** domain $= \{x|x \in R, x \neq 0\}$, since $xy = 2$ becomes $y = \frac{2}{x}$ if we solve for y.

Review exercises

1. $\frac{4x^8}{y^6}$ **2.** x^6 **3.** y^4 **4.** $x^{y+1} \neq x^y \cdot y$ since the bases, x and y, are not the same **5.** $2(x + 2y)(x - 2y)$ **6.** $3\sqrt{6}$ units **7.** 60 m by 120 m

Exercise 10-2

Answers to odd-numbered problems

1. $f(0) = -2$; $(0,-2)$ **3.** $f\left(\frac{2}{3}\right) = 0$; $\left(\frac{2}{3},0\right)$ **5.** $g(7) = 58$; $(7,58)$ **7.** $g\left(\frac{1}{2}\right) = -\frac{15}{4}$; $\left(\frac{1}{2},-\frac{15}{4}\right)$ **9.** $f(a + 1) = 3a + 1$; $(a + 1, 3a + 1)$ **11.** $f(a^2) = 3a^2 - 2$; $(a^2, 3a^2 - 2)$ **13.** 27 **15.** 16 **17. a.** $4x^2 + 8hx + 4h^2$ **b.** $8hx + 4h^2$ **c.** $8x + 4h$ **19. a.** $2x^2 + 4hx + 2h^2 + 3x + 3h + 2$ **b.** $4hx + 2h^2 + 3h$ **c.** $4x + 2h + 3$ **21. a.** $5 - 2x - 2h$ **b.** $-2h$ **c.** -2 **23.** $f(-5) = -17$, $(-5,-17)$; $f(0) = -2$, $(0,-2)$; $f\left(\frac{2}{3}\right) = 0$, $\left(\frac{2}{3},0\right)$ **25.** $h\left(-\frac{1}{2}\right) = \frac{11}{4}$, $\left(-\frac{1}{2},\frac{11}{4}\right)$; $h(0) = 1$, $(0,1)$; $h(3) = 22$, $(3,22)$ **27.** $g(-15) = 10$, $(-15,10)$; $g(0) = 10$, $(0,10)$; $g\left(\frac{6}{5}\right) = 10$, $\left(\frac{6}{5},10\right)$ **29.** $48x^2 + 24x - 2$ **31.** $27x^4 - 90x^2 + 70$ **33.** 7 **35.** −6 **37.** (14,−10), (32,0), (212,100) **39.** (1,1), (3,27), (5,125); domain $= \{s|s > 0\}$ **41.** (2,49), (3,69), (5,109)

Solutions to trial exercise problems

9. $f(a + 1) = 3(a + 1) - 2 = 3a + 3 - 2 = 3a + 1$; $(a + 1, 3a + 1)$ **12.** $f(5) = 3(5) - 2 = 15 - 2 = 13$ and $f(2) = 3(2) - 2 = 4$. So $f(5) - f(2) = 13 - 4 = 9$.

19. a.
$$\begin{aligned} f(x + h) &= 2(x + h)^2 + 3(x + h) + 2 \\ &= 2(x^2 + 2xh + h^2) + 3x + 3h + 2 \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h + 2 \end{aligned}$$

b.
$$\begin{aligned} f(x + h) - f(x) &= (2x^2 + 4xh + 2h^2 + 3x + 3h + 2) \\ &\quad - (2x^2 + 3x + 2) \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h + 2 - 2x^2 \\ &\quad - 3x - 2 \\ &= 4xh + 2h^2 + 3h \end{aligned}$$

c.
$$\begin{aligned} \frac{f(x + h) - f(x)}{h} &= \frac{4xh + 2h^2 + 3h}{h} = \frac{h(4x + 2h + 3)}{h} \\ &= 4x + 2h + 3 \text{ since } h \neq 0 \end{aligned}$$

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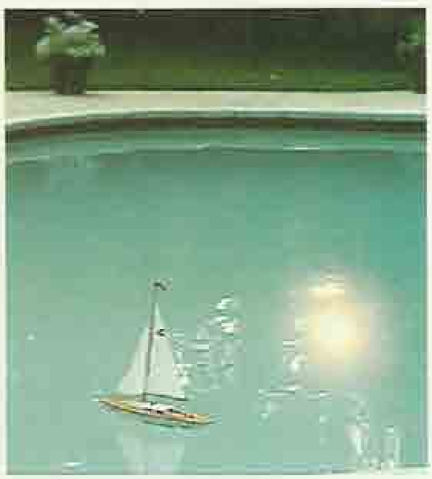

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